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G. S. Golitsyn

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16. Abstract

A new general theory is described of circulation in planetary atmospheres, one developed on the basis of consistent use of similarity and dimensional arguments supplemented by energy and thermodynamic considerations. The causes and nature of motions of various scales are discussed, observational, numerical, and laboratory methods of studying meteorological fields are described, and a survey is given of astronomical and atmospheric parameters for various planets of the Solar System.

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G. S. Golitsyn

Foreword /3*

The development of research on the planets of the Solar System by terrestrial and space methods faces meteorologists with the imperative demand for answers to such questions as the vertical and horizontal distribution of temperature in the atmosphere of a planet, evaluation of wind speeds and wind distribution in space and in time, the structure of wind gusts and wind distribution in the boundary layer of the atmosphere, and a large number of other questions. Of decisive importance among these problems is that of the general circulation as the most complex and least understood aspect. If this problem could be solved to a certain degree of approximation, it would be possible, by use of the experience acquired in geophysical hydrodynamics, to assess with a certain amount of confidence many characteristics of smaller scale movements.

An excellent account of the status of the general circulation problem and its difficulty for the Earth's atmosphere has been given in the book by Lorenz (1967). The theory of general circulation represents the foundation for long-range weather forecasting and a complete theory of climate.

The basic working method of this theory is represented by numerous experiments on simulation of the behavior of the atmosphere in the case of predetermined supplies of heat. This method also began to be applied for study of circulation of the atmospheres of other planets, specifically, Mars and Venus.

However, like all experimental methods, numerical experiments, which still require a very high expenditure of labor and resources, do not yield a general approach to the problem. At the same time, they hold out too little hope for the construction of even a highly simplified but general analytical theory of circulation (see Lorenz, 1967), especially if an effort is made to describe the structure of circulation in space and time. But if we confine ourselves to an endeavor to obtain merely certain average universal characteristics of the general circulation, we can be confident of success in this direction by using the general methods of the theory of similarity and dimensionality. And such a theory as this was elaborated by the author at the end of the 1960s and the beginning of the 1970s. It provides the possibility of classifying the circulation of planetary atmospheres, and in a number of instances of obtaining important specific results, in a natural fashion and exclusively on the basis of external astronomic factors and properties of the atmospheres of planets.

The greater part of this book is devoted to presentation of this theory, /4
which by now has undergone a certain amount of refinement, and to its application

*Numbers in the margin indicate pagination in the foreign text.

to various planets and to the atmosphere of the Sun. Wherever possible, the findings of the theory are compared with the results of observations and numerical experiments.

Knowledge of the characteristics of general circulation, and of the mean wind in particular, permits assessment also of the structure of the boundary layer on Mars and Venus on the basis of the general theory of similarity developed by A. M. Obukhov, A. S. Monin, and their associates. Evaluations of the characteristics of the turbulent fluctuations of the velocity and temperature fields in the atmospheres of other planets are naturally also obtained, this being a factor of importance for certain technical applications. Hence consistent utilization of the methods of the theory of similarity and dimensionality are typical of virtually all the questions considered in the book, with the exception of the problem of the general dust storms on Mars, which is characterized by a somewhat different method of investigation. As we know, the enormous dust storm on Mars which began in September of 1971 and did not end until the middle of January of 1972, occurred during the period of exploration of Mars by means of the Mars-2, Mars-3 and Mariner-9 space stations. Considering the importance of the question, and to draw the attention of meteorologists to this extremely difficult and highly interesting problem, the author believed it to be necessary to include in the book a brief survey of the data gained in observations and considerations of his own regarding the origination, development, and attenuation of the dust storms.

The hydrodynamics of planetary atmospheres is undergoing rapid development, attracting broad interest on the part of meteorologists, specialists in atmospheric physics, astronomers, and space scientists. An increasing number of scientific publications are making their appearance. For this reason a bibliography of the works which the author believes to be the most important ones that appeared in print by spring of 1972 is given in the book.

The title of the book may appear to be too comprehensive, inasmuch as many questions of atmospheric dynamics are discussed here only in brief or are not touched on at all. However, if the reader is familiar with conventional hydrodynamics, the material presented may in reality be regarded as an introduction to the dynamics of planetary atmospheres, since we provide an analysis of the simplest and so to speak most elementary properties of the dynamic equations and arrive at our findings exclusively by use of the theory of similarity and dimensionality. Analytical and numerical calculations of the space-time picture of atmospheric flows obviously represent the next logical step in investigation of the motions of planetary atmospheres, although the inner logic of presentation of a subject does not always coincide with the actual stages of development of the science itself, and the elementary analysis described here of general circulation was made much later than the many numerical experiments and model analytical calculations.

All the necessary theoretical information is given, at least in concise form, in the text itself, although the reader familiar with hydrodynamics, say, /5
to the extent of the first few chapters of the book by L. B. Landau and Ye. M. Lifshits, Mekhanika Sploshnykh Sred [Mechanics of Solid Media], and with the

first two chapters of the book by L. I. Sedov _Metody Podobiya i Razmernosti v Mekhanika_ [Similarity and Dimensionality Methods in Mechanics] will be more accustomed to the manner of exposition adopted in this book.

I am deeply grateful to my teacher in physics of the atmosphere in the broad sense of this term, A. M. Obukhov, who so early as 1964 suggested to me that I concern myself with the atmospheres of other planets. He pointed out on many occasions that the study of other planets will enable us to obtain a deeper and better understanding of the laws governing the behavior of the Earth's atmosphere. This keen interest, the always fruitful discussions held with him, and his advice contributed toward deepening of my understanding of the subject and has inspired me to continue my work. I am also greatly obliged to A. S. Monin, A. M. Yaglyy, B. I. Tatarskiy, L. A. Dikiy, Ye. A. Novikov, A. S. Gurvich, and S. S. Zilitinkevich, association with whom, and often joint work as well, fostered clarification of particular questions. A separate word of thanks is due to B. I. Moroz, many discussions with whom contributed toward my gaining a thorough understanding of the subject of physics of planetary atmospheres and grasping the tasks with which this field of science is faced.

I must express my thanks to the many students attending my lectures at various symposia, conferences, seminars, and lectures, who by their questions, perplexity, and criticism compelled me again and again to devote further thought to the logic of the theory and the manner of its presentation. This process contributed greatly to the development and thorough substantiation of the theory itself. Among these most critical, but constructively critical students, I should like to make special mention of Dzh. Charni, N. A. Phillips, and F. P. Brotherton. I am also grateful to R. M. Goody, B. A. Smith, and S. I. Rasool, who kindly sent me magnificent photographs of planets.

The editor of the book, F. V. Volzhanskiy, has done much constructive work on the book which has contributed greatly to improvement in presentation of the material.

G. S. Golitsyn

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1. Brief History of the Study of Motions in Planetary Atmospheres

Winds blow in the atmospheres of planets. From the most ancient times man has observed in the Earth's atmosphere a wide diversity in winds and their association with certain seasons, times of the day, and types of weather. The vigorous development of navigation and the age of great geographic discoveries in the 16th and 17th centuries led to the idea that there are more or less permanent wind systems on Earth: tradewinds in the subtropics and tropics and westerly winds in the temperate latitudes. The first attempt to arrive at a scientific understanding of the causes of the occurrence of winds in the atmosphere was made by the well-known English astronomer Edmund Halley (1586). Halley sought to explain the tradewinds by the diurnal variation in the maximum heating of the atmosphere, this variation following the Sun from east to west. A work which played an enormous part in the history of meteorology, and in particular in the theory of general circulation was an article by Hadley (1735) in which for the first time in the history of science, almost 100 years before introduction of the Coriolis force bearing his name, in which it was pointed out that the unevenness of heating of the surface of the planet represents the main reason for the occurrence of winds, and that the rotation of the Earth has a decisive influence on the nature of their distribution. In the 19th century a number of ideas were advanced regarding the nature and structure of general circulation, ideas associated with the name of Ferrel and others (a good survey of the history of development of ideas regarding the circulation of the Earth's atmosphere is given in a book by Lorenz (1967)), and in this same century hydrodynamics begin to move into the sphere of meteorology. The present-day status of the problem will be discussed in Section 3, but for the time being we will go on to other planets.

The existence of winds or atmospheric flows on other planets was first discovered on Jupiter and Saturn. These winds are identified on the basis of the periods of rotation of individual clearly different spots on the disc of the planet. The first pertinent observations and measurements were apparently made by Cassini around 1690. The history of these observations, together with detailed summaries of the latter, are discussed in the books by Peek (1958) on Jupiter and Alexander (1962) on Saturn. A particularly large number of obser- /7
vations and measurements of the periods of rotation for both planets was made by the English amateur astronomer Williams during the last 25 years of the 19th century. It was he who introduced the term "atmospheric flows" to explain the causes of difference in the periods of rotation at different latitudes.

The flows on both planets are rigidly zonal, that is, they run along the parallels. No one has ever observed any appreciable systematic meridional components. The apparent discs are characterized by a system of dark and light zonal bands, which will be discussed in greater detail later in the appropriate sections. In this respect they are also resembled by Uranus, on which a slight zonal structure is observed. A typical feature of the giant planets is represented by the higher speeds of rotation of the equatorial regions than the speeds of rotation of the temperate latitudes. On Jupiter the relative speeds of rotation are of the order of 100 m/sec, and on Saturn 400 m/sec in comparison to the mean speeds of the temperate latitudes.

So early as 100 years ago there was systematic if relatively rare observation, no more than a few times a year, of clouds on Mars. These clouds tend to appear in specific places on the planet, that is, they are probably somehow associated with the surface relief of the planet. At times they become almost stationary, especially the white clouds. Yellow clouds, the speed of movement of which is around 10-20 m/sec or more, are less frequently observed. The latter are associated with sand or dust storms. In some cases such storms cover virtually the entire apparent disc of the planet like a mist, as for example in 1924, 1956, and 1971[1] (see Moroz, 1967).

Lastly, motion was discovered in the atmosphere of Venus in our own days, in the 1960s. It is the so-called 4-day circulation of ultraviolet clouds. It was first observed by Boyer and Camichel (1961, 1965, 1967). Extensive observations were conducted by Smith (1967). Dark details rotating with a period of around 4 days in the same direction as that of the planet itself can be seen in ultraviolet light. Their speed relative to the surface is on the order of 90-110 m/sec. The altitude of these clouds is around 100 km, at which the pressure is of the order of 1 mb, that is, they represent a stratospheric or even mesospheric effect. Indirect evidence of the existence of certain probably small-scale movements of the turbulent type deep in the atmosphere of Venus as well is represented by the fact that radio signals passing through the atmosphere of the planet undergo random fluctuations which /8
may be interpreted as being due to turbulence (see Section 18). Such fluctuations were recorded with the signals both of Soviet and of American (Mariner-5) space stations which descended into the atmosphere of the planet.

The Soviet space stations of the Venus series made it possible to conduct the first direct measurements of wind speed and turbulent gusts in the places at which the stations descended into the atmosphere of the planet, on the basis of the Doppler shift of the transmitter frequency (Kerzhanovich et al., 1969; Kerzhanovich, 1972; Kerzhanovich et al., 1972). This procedure is described in Section 14.

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[1]A large body of material of a directly synoptic nature has been obtained by Mariner-9 by means of television and infrared interferometer-spectrometer (for the preliminary results see Hanel et al., 1972). The data of the latter make it possible to establish the vertical profiles of temperature in the atmosphere of Mars, on the basis of which it is also possible to establish the wind field structure. In the cold hemisphere of Mars, one can easily detect in the many television photographs obtained during the dust storm and after it cloud formations of the cyclone and frontal type, banks of undulated clouds beyond mountains, and convection clouds. By the time work on this book was completed (1972), all of this enormous body of material was still largely unprocessed and unpublished. Several photographs of Mars obtained from Mariner-9 are given in Section 13, which is devoted to Mars in particular.

For Mercury alone among the major planets, atmospheric motions have yet to be observed. In addition, there is as yet no convincing evidence even of the existence of an atmosphere on this planet; only the upper limits of estimates of the mass of its atmosphere are available.

The observations thus show that the atmospheres of the planets are in movement. The structure of their apparent discs unquestionably reflects the nature of the atmospheric motions. Photographs of the planets are given in Figure 1. Only Uranus, Saturn, and Jupiter are similar, in exhibiting zonality of motion. Venus has no structure whatever in visible light, only light seas /9
and dark continents are visible on Mars owing to the thinness of the atmosphere and the rare appearance of clouds, and the details of the cloud structure of Earth display no regularity whatever.

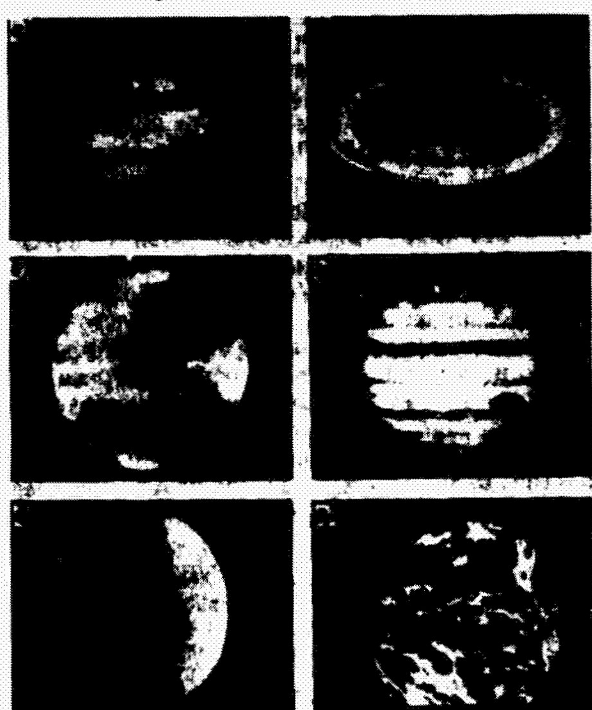

Figure 1. Planets of the Solar System: a, Uranus (Sketched by Antoniadi); b, Saturn; c, Mars; d, Jupiter; e, Venus; f, Earth (Photography from a Point Near the Moon). The photograph was kindly sent by R. M. Goody.

2. Causes and Nature of Atmospheric Motions (With the Earth's Atmosphere as an Example)

The chief cause of the motions of the atmosphere is represented by the uneven progress of thermal energy to various parts of the latter and uneven heating of the atmosphere. In the presence of the force of gravity, mechanical equilibrium is impossible in a fluid unevenly heated in the horizontal plane, and the Archimedes forces of buoyancy that arise lead to the occurrence of motions which tend to mix the fluid (Landau, Lifshits, 1954). The general circulation of the atmosphere also occurs for this reason. In particular, in the Earth's atmosphere local winds of the monsoon or sea breeze type are formed as a result of the circumstance that air is alternately warmer above land or above the sea. Cumulus convection in summer is caused by intense overheating of land and disruption of the vertical adiabatic stability, and in winter above oceans by the intrusion of cold air masses from continents.

Planetary atmospheres are open systems that receive energy from outside, from the Sun, and release excess energy by way of longwave radiation into outer space. A energy balance is achieved only for the atmosphere in general, while there is no balance between the arrival and departure of energy for each individual region of the atmosphere, and this circumstance continuously sustains motions in the atmosphere. The atmospheric motions themselves play a vital part in determining the nature of this balance, since they ultimately transfer heat from the more greatly heated regions to the cooler ones.

Taking the example of the Earth's atmosphere, let us determine what are the spatial scales of the motions, or to employ the current meteorological terminology, what is the spatial spectrum of these motions, that is, what kinetic energy is carried by motions of a specific scale. Analysis of this spectrum enables us to introduce certain useful concepts often employed later, and to make a rough classification of the motions into categories according to their spatial scales. There is no reason to expect that the spectra of motions on other planets will in their details or even in general outline be similar to the spectrum of motions of the Earth's atmosphere, but some sections of this spectrum are universal and should be manifested everywhere in one form or another.

The spatial spectrum of the velocity field in the Earth's atmosphere has been studied in more or less detail (both in theory and on the basis of observational data) only in two extreme regions: for the largest scales with space wave numbers k from 0 to 20, that is, for the zonal component of wind in scales from constant zonal flow (k = 0) and a wavelength of $2\pi r$ (k = 1, r = /10
= radius of planet) approximately to 2000 km, and for the smallest scales, of the order of 10 km and less. This circumstance is due to the nature of the observational data and to the concept itself of spatial spectrum. To construct this spectrum (see Monin, Yaglom, 1967) it is necessary to have knowledge of a large number of instantaneous observations, that is, instantaneous pictures of the velocity field over the entire globe without any gaps in space. On the basis of this field there is calculated correlation function

$$B_{ik}(\mathbf{r}_1 - \mathbf{r}_2) = \overline{[v_i(\mathbf{r}_1) - v_i(\mathbf{r}_2)][v_k(\mathbf{r}_1) - v_k(\mathbf{r}_2)]}, \qquad (2.1)$$

in which r_1 and r_2 are points of observation and v_i and v_k are the velocity components; the line above the righthand term denotes averaging.

For a spatially homogeneous and isotropic random velocity field, correlation tensor function $B_{ik}(r_1 - r_2)$ is invariant relative to transfers and rotations in space of vector $r = r_1 - r_2$ and depends only on the modulus of the latter, $r = |r|$, while the Fourier transform of the correlation function assigns the spectrum for spectral density

$$F_{ik}(\mathbf{k}) = \frac{1}{8\pi^3}\int\int\int B_{ik}(\mathbf{r})\, e^{i\mathbf{k}\mathbf{r}}\, d\mathbf{r}. \qquad (2.2)$$

In this instance tensor $F_{ik}(k)$ for an incompressible fluid is defined by a single scalar function — the spectral density of energy E(k), integral $\int_0^\infty E(k)\,dk$ equalling the total kinetic energy of the unit mass of the fluid.

The velocity fields in the atmosphere may be considered to be uniform and isotropic only for fairly small scales, of the order of 1 km and less. Larger scales may be regarded in approximation as such only in horizontal planes. In study of the largest (global) scales data are usually taken along a circle of latitude, and the velocity field is considered to be uniform along it. The concept of spatial spectrum (in the latter case determined for one latitude) is nevertheless found to be useful for fields of such restricted uniformity.

We begin with the region of small scales in which the laws established in 1941 by Kolmogorov-Obukhov are valid. A. M. Kolmogorov (1941) took as his basis considerations of the theory of similarity and dimensionality, and A. M. Obukhov (1941) considerations of a model nature, but they arrived at identical results. We will have need of the conclusions of this theory more than once in what follows; in addition, the Kolmogorov theory is a simple and convenient case for demonstration of the use of general methods of the theory of similarity and dimensionality.

Kolmogorov formulated his theory on the basis of two hypotheses of a physical nature concerning the structure of the velocity field in turbulent flow at large Reynolds numbers $Re = UL/\nu$, in which U is the characteristic flow /11
velocity, L the so-called external turbulence scale, or scale of basic energy bearing vortices, in which energy is introduced into the flow, and ν is the kinematic viscosity. If $Re >> 1$, the largest vortices are unstable and generate smaller vortices, transmitting their kinetic energy to the latter. If the Reynolds number for these vortices is also large, they again prove to be unstable and again are broken down into smaller ones, and so forth. The process of breakdown of vortices into increasingly smaller ones down to the very smallest, for which the corresponding number is $Re \approx 1$, that is, viscous dissipation becomes substantial, was described qualitatively by Richardson (1922). A. M. Kolmogorov embodied these concepts in concrete form by introducing his first similarity hypothesis, which states that, at sufficiently large Reynolds numbers, there must exist a region of scales r much smaller than external scale L, on which turbulent vortices will be uniform and isotropic, and the nature of the turbulence will be determined by kinematic viscosity and quantity ε, the speed of transfer of kinetic energy (per unit mass) along the series of vortices from the larger to the smaller. In the stationary state quantity ε will equal the rate of dissipation of kinetic energy to heat owing to the action of viscosity on the smallest scales. For this reason ε is often termed simply the dissipation rate, or even merely dissipation.

The magnitude of the scales at which viscous dissipation occurs may be found from considerations of dimensionality. By definition dimensionality ε is $\frac{\text{energy}}{\text{mass}\cdot\text{time}}$, or $[\varepsilon] = cm^2/sec^3$. The dimensionality of kinematic viscosity $[\nu] = cm^2/sec$. From these two values one can by an unequivocal process formulate the value having the dimensionality of length:

(2.3)

Quantity l_0 is termed the Kolmogorov microscale. Since we also have spatial scale r, from the latter and l_0 we can formulate dimensionless length r/l_0 on which alone all the spatial characteristics of turbulence should depend (when $r \ll L$).

Kolmogorov further noted that in the intermediate region of scales

$$l_0 \ll r \ll L \tag{2.4}$$

viscosity no longer exerts any effect, but the vortices are uniform and isotropic, or more precisely are locally uniform and locally isotropic. The word "locally" is to be understood to mean the uniformity and isotropicity of the statistical characteristics of the differences of two quantities in turbulent flow taken at distance r. Hence for values r satisfying condition (2.4) the structure of the flow is defined by unique external dimensional parameter ε. This makes up the content of Kolmogorov's second hypothesis. Since over the interval of scales of (2.4), only forces of inertia are active, owing to which /12
energy is transferred over the spectrum of vortices from larger ones to increasingly smaller ones, this interval is often termed the inertial interval, and the entire interval $r \ll L$ is called the equilibrium interval.

No dimensionless combination whatever may be constructed from ε and r, but it is possible to formulate a quantity having the dimensionality of velocity or the square of velocity. As the statistical characteristic of locally uniform and locally isotropic turbulent flow, Kolmogorov proposed the use of the so-called structural function, the root mean square difference of two quantities characterizing the flow at points 1 and 2, the distance between which equals r. Inasmuch as the velocity is a vectorial quantity, we take the projections of the velocity in direction r, that is, its longitudinal components. The longitudinal structural function of velocity then depends only on modulus r and is defined as

$$D_{ll}(r) = \overline{[v_l(\mathbf{r}_1) - v_l(\mathbf{r}_2)]^2} = \overline{[\Delta v_l(\mathbf{r})]^2}. \tag{2.5}$$

Since D_u depends only on r, the unique quantity having the dimensionality of the square of velocity which may be formulated from ε and r is the following:

$$D_{ll}(r) = C\varepsilon^{2/3} r^{2/3}, \tag{2.6}$$

in which C is a certain constant. The rule set forth in (2.6) has been confirmed experimentally by numerous measurements in the laboratory, the atmosphere, and the ocean (see Monin, Yaglom, 1967, Section 23), which indicate the dimensionless constant to be $C \approx 2$.

A rule equivalent to (2.6) for spectral density was established by Obukhov (1941):

$$E(k) = C'\varepsilon^{2/3} k^{-5/3}, \tag{2.7}$$

in which constant C' is associated with C and equals approximately 0.5 (see Monin, Yaglom, 1967, Section 23).

The region of applicability of the theory of similarity in which the dimensionless criteria of similarity (in our case r/l_0) are much larger (or much smaller) than unity is termed the region of similarity, since, as follows from physical considerations, in this region not every external parameter entering into the dimensionless criterion is essential (in this case ν). Then for the quantities with which we are concerned we can write unique formulas in the form of algebraic monomials by using the remaining external parameters, this meaning that the quantities in question will be similar or even identical if the scales are properly selected.

Now let us return to the largest scales. Since the atmospheres of planets are thin spherical envelopes, the spectrum for them is determined by development not into an integral but into a Fourier series, say on the basis of a circle of latitudes (or on the basis of spherical functions). Large disturbances with scales much larger than the altitude of the uniform atmosphere are quasi-two-dimensional, that is, the vertical velocity components are much smaller than the horizontal ones. /13

Considerations regarding the form of the energy spectrum in two-dimensional flow were first advanced by Batchelor at the beginning of the 1960s, but the corresponding work was not published by him until 1969. Similar considerations were also published by Kraichnan (1967). The cascade process of transfer of energy from large vortices to smaller ones cannot take place in a rigorously two-dimensional fluid (Lee, 1951; Fjortoft, 1953; Kraichnan, 1967), owing to absence of the effect of extension of the vortical filaments. This follows directly from preservation of the square of the vortex in a two-dimensional nonviscous fluid. However, a similar cascade process of transfer over the spectrum may occur for a quantity termed entrophy (Kraichnan, 1967), which is introduced like ε, as

$$\eta = \frac{1}{2}\frac{d\overline{\Omega^2}}{dt} = \frac{1}{2}\nu\overline{\left(\frac{d\Omega_i}{dx_k} + \frac{d\Omega_k}{dx_i}\right)^2}, \qquad (2.8)$$

in which Ω_i is the vortical velocity vector component. Then from the quantities $\eta\,[\mathrm{sec}^{-3}]$ and $k\ [\mathrm{cm}^{-1}]$, which define the structure to two-dimensional turbulence over a certain scale interval, one can formulate a quantity having the dimensionality of the spectral density of energy:

$$E(k) \sim \eta^{2/3} k^{-3}. \qquad (2.9)$$

Real large-scale atmospheric motions are actually three-dimensional, but, for planets rotating with sufficient speed, they are quasigeostrophic, that is, the Coriolis force is approximately in balance with the pressure gradient. In this instance the so-called potential vortex is preserved in the flow (Ertel, 1942; also see Monin, 1968, 1969), and the concept of potential entrophy may be introduced. Considerations of dimensionality then also lead to formula

(2.9), but η is now the potential entrophy (see Charney, 1971; Gavrilin et al., 1972).

Processing of experimental data on the wind field in the temperate latitudes (see the surveys by Leith, 1971; Gavrilin et al., 1972) has yielded the relation $E(k) \sim k^{-3}$ for the spectral region k = 7 to 20.

For k = 6 to 7 the energy spectrum usually exhibits a maximum, and a small decrease (or approximately constant spectrum) closer to zero. There apparently are no universal laws for this region on the largest scales.

The existence of a synoptic maximum for $kr \approx$ 6 to 7 is to be ascribed to the so-called baroclinic instability effect, which is the fact that the excess potential energy of the atmosphere is converted to the kinetic energy of vibrations with wavelengths corresponding to the wave numbers indicated. Precisely these modes of movement are responsible for the formation of cyclones and anticyclones in the atmosphere, that is, weather as it is ordinarily understood (see Lorenz, 1967).

The scales of motions between 1000-2000 km and approximately 20 km are among the least studied regions of the spectrum. The reason is by the sparse network of weather stations; the average distances between them are several hundred kilometers, so that it is not possible to construct spatial spectra directly on the basis of observational data. /14

If the frequency spectra at one point are known, an attempt may be made to establish the spatial spectra by means of the relation $k = \omega/U$ (ω is the circular velocity of time pulsations, and U the mean wind speed), that is, the Taylor hypothesis stating that the evolution in time of a vortex having wave number k is much longer than the time of passage of this vortex by an observer. A procedure such as this employed by Ellsaesser (1969) in the processing of a large body of observational material demonstrated that for periods of up to 6 hours the structural function of velocity in time is

$$D_v(t) \approx C_1 (\varepsilon U t)^{2/3}.$$

This relation is also satisfied for periods of 6 to 36 hours for structural functions constructed on the basis of data for the entire northern hemisphere, with the exception of the region of the tropic stratosphere. If a mean wind speed of $U \approx 10$ m/sec is adopted, the "two-thirds law" is valid for scales up to $r = Ut = 10$ m/sec·36·3600 sec $\approx$ 1300 km. Of course, for scales of the order of hundreds of thousands of kilometers, the validity of application of the Taylor hypothesis is always doubtful, and generally speaking it does not follow from proportionality $D_v(t) \sim t^{2/3}$ that $D_v(r) \sim r^{2/3}$ and $E(k) \sim k^{-5/3}$. However, the error will apparently not be too large if the relation $E(k) \sim k^{-5/3}$ is assumed in this region of the spectrum (the wave number interval being one and one-half or two decades). At any rate, this may be the top estimate.

Yet another circumstance which it is the simplest to understand precisely from the viewpoint of the $k^{-5/3}$ law is represented by the results of Richardson (1926). Richardson was the first to demonstrate that simple, although highly unusual statistical relationships obtained in the atmosphere. After assembling and processing a great variety of data, Richardson discovered that the root mean square distance between two particles labeled in some way in the atmosphere is proportional to the cube of time from the beginning of observation of these particles[2]:

$$\overline{r^2} \sim t^3, \tag{2.10}$$

this relationship remaining valid up to distances of the order of 1,000 km. The coefficient of relative diffusion of these particles may be defined as

$$K = \frac{d\overline{r^2}}{dt} \sim t^2 \sim (\overline{r^2})^{2/3} \sim \bar{r}^{4/3}. \tag{2.11}$$

This is the well-known Richardson law, according to which the coefficient of relative diffusion of two particles is proportional to the mean distance between them to the 4/3 power. /15

Theoretical explanation of the Richardson law was given by Obukhov (1941), in the same work in which the law represented by (2.7) was established. It is based on analogous considerations of similarity and dimensionality, although its applicability to such large scales remains uncertain. If ε is assumed to be a unique parameter defining the structure of turbulent flow, it is possible to construct from ε and r a unique combination having the dimensionality of diffusion coefficient

$$K = C_2 \varepsilon^{1/3} r^{4/3}. \tag{2.12}$$

The Richardson-Obukhov law of (2.12) has been repeatedly verified in the atmosphere (Monin, Yaglom, 1967, Section 24) and in the ocean (Okubo and Ozmidov, 1970). According to a number of estimates, constant C_2 is on the order of 0.1. If quantity ε is in any way known for large-scale motions, formula (2.10) is used to estimate the coefficient of large-scale mixing of the atmosphere (Zilitinkevich, Monin, 1971).

It is also possible to employ formula (2.10) to estimate the relative speeds of divergence of two particles

$$\Delta v = \frac{d(\overline{r^2})^{1/2}}{dt} \approx t^{1/2} \sim \bar{r}^{1/3}.$$

The expression obtained for Δv coincides in form with the results of Kolmogorov (compare (2.6) with (2.5) being taken into account), although, strictly speaking, it is not equivalent to it (in this context compare the discussion of

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[2]It is to be noted that the statistical relationships of the wandering of a Brownian particle known up to that time yielded $\overline{r^2} \approx \nu t$.

the relationship between the Lagrangian and Eulerian descriptions of turbulence given by Monin and Yaglom (1967, Section 24)).

A diagrammatic representation of the space spectrum of atmospheric movements is presented in Figure 2. The ordinates are given in conventional units, and the abscissa in dimensionless units kr, in which r is the radius of the Earth. The boldface dot on the ordinate axis marks the energy corresponding to $k = 0$, that is, purely zonal flow. Dots indicate the first ten harmonics, and a solid line is drawn after them. Synoptic maximum $kr \approx 6$ is followed by a region of quasi-two-dimensional turbulence, in which $E(k) \sim k^{-n}$, and $n \approx 3$. The latter is followed by a region of large-scale Richardson diffusion, in which the relation $E(k) \sim k^{-5/3}$ is proposed as the upper estimate. In the region of scales of the order of tens of kilometers, there is a mesometeorological minimum followed by a maximum of small-scale turbulence due to instability of the wind, convection, and the like, and lastly the longest, purely Kolmogorov region of the spectrum (measured in decades).

What typical features of the space spectrum of the Earth's atmosphere are inherent in the atmospheres of other planets? A synoptic maximum must apparently be preserved for planets that rotate at not too great a speed. As a matter of fact, the numerical experiments of Leovy and Mintz (1966, 1969) on simulation of the general circulation on Mars indicate that such a maximum could be observed at $kr = 4$. In the case of large and rapidly rotating planets, this maximum cannot exist, since the large-scale motions observed exhibit no apparent instability, or they are greatly displaced in the region of large kr values. It is to be expected that a second (small-scale) maximum always exists, to the extent that it is associated basically with local instabilities of the wind in the atmosphere on scales of the order of the thickness of the atmosphere. /16

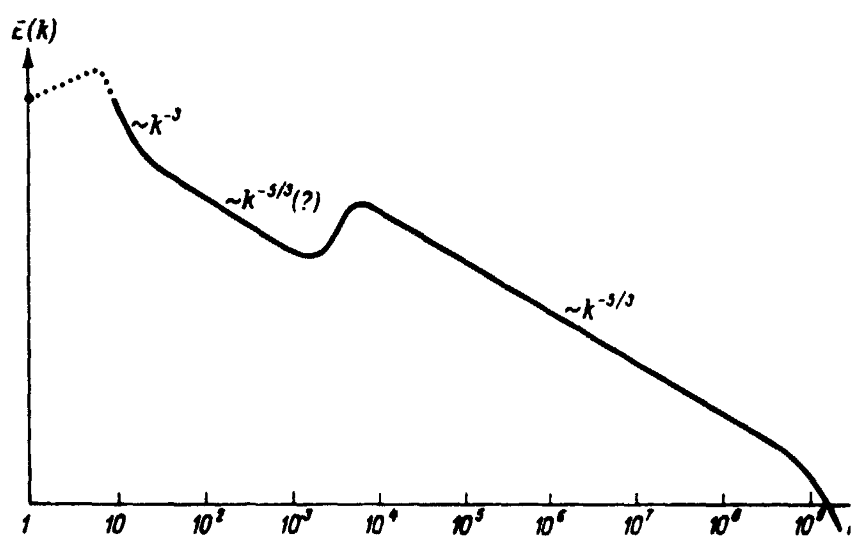

Figure 2. Diagram of the Energy Spectrum of Atmospheric Motions over a Broad Interval of Wave Numbers.

3. Methods of Studying the Dynamics of Planetary Atmospheres

The role of atmospheric motions in atmospheric dynamics varies in keeping with the various space and time scales of the motions, and the methods of studying them vary as well. Let us consider the methods employed to study the dynamics of the Earth's atmosphere.

They include primarily observations of temperature, wind, pressure, humidity, cloudiness, and radiation which are conducted regularly at thousands of weather stations very unevenly distributed over the Earth's surface. There is a sufficient density of stations only over a small part of the land areas (around 20%), while virtually no provision is made for observations over the remainder of the surface, represented chiefly by the oceans. This is one of the main reasons for the poor quality of long-term weather forecasts at the present time. For this reason meteorologists throughout the world are making /17
preparations to carry out a World Program of Global Atmospheric Process Research (PIGAP) which is to be conducted during the years 1976-1977. The basic information, including the temperature of the underlying surface, the profiles of temperature and humidity in the atmosphere, observations of winds on the basis of displacement of clouds and the drift of balanced balloons, etc., will be obtained from satellites.

The technique of using satellites may also be fully applied for meteorological observations of the atmospheres of other planets. Observations of radiation escaping from the atmosphere of Mars in the CO_2 absorption region in the vicinity of 15 μ have yielded highly interesting results on the vertical temperature distribution of the Martian atmosphere (Hanel et al., 1972). For the time being, however, it is realistic to expect that only individual experiments may be conducted.

Another approach, one replacing observations in significance, is represented by numerical simulation of the behavior of planetary atmospheres. In theory it is possible in this way to simulate dynamic processes of any scale. Simulation is especially useful for study of the general circulation and climate on other planets. At the present time this is basically the only economical method of studying the detailed structure of the distribution of wind and temperature, although it still requires the consumption of a very great amount of labor. Starting with the first numerical experiments of Phillips (1956), such experiments and, by now more than 30 of them are known, have reproduced fairly well the basic features of the general circulation of the Earth's atmosphere. Leovy and Mintz (1966) were the first to apply this method for study of the circulation of another planet, specifically, Mars. They published detailed calculations in 1969. Under the guidance of A. S. Monin (see Zilitinkevich, Monin, et al., 1971; Turikov and Chalikov, 1971; Chalikov et al., 1971) numerical experiments were carried out in 1970-1971 on simulation of the circulation of the Venusian atmosphere.

However, a basic defect is inherent in the numerical experiments, as in all other experiments or observations. They show how a phenomenon develops or

takes place, but do not explain it. Thorough understanding of the essence of a phenomenon requires a full series of controlled experiments performed at different values of one or more parameters defining the phenomenon being studied. Variation of particular parameters characterizing an atmosphere is especially necessary in study of the circulations of other planets, since the values of many of them are not known with sufficient accuracy. For the sake of greater certainty regarding the basic features of the predicted circulation, it is necessary to know that the variations in these parameters is insignificant. For example, in numerical simulation it is necessary to specify with high precision the optical properties of the absorbing atmospheric gases. Very little study has been devoted to these properties at high temperatures and pressures. Precisely for this reason it was necessary for the authors of the experiments in question on simulation of the circulation of the Venusian atmosphere to perform two experiments: in the first of the experiments, the /18
bulk of the solar radiation assimilated by the planet reached its surface, and in the second one, it was entirely absorbed in the upper theoretical layer of atmosphere.

Essential information on the nature of the circulation of the Earth's atmosphere was obtained in laboratory studies of the convection of a rotating and not uniformly heated fluid. Baroclinic instability, the formation of cyclonic and anticyclonic vortices, the formation of fronts and jet streams, processes of transition of energy from the potential to the kinetic form, transfer of energy over the spectrum, and other important features of the general circulation of the atmosphere are reproduced in laboratory experiments (Lorenz, 1967; Starr, 1968; Monin, 1969; Hide, 1970). At the same time it is quite obvious that many seemingly highly important aspects of the atmosphere and its circulation cannot be simulated in small laboratory units. Hence the success achieved in laboratory experiments permits the conclusion that the role of such aspects under natural conditions is not a decisive one in determining the basic features of the general circulation of the Earth's atmosphere. This is the most significant result of laboratory experiments. We shall cite four basic conclusions (Hide, 1970).

1. In the Earth's atmosphere the ratio of the vertical scale of movement to the horizontal is approximately 10^{-3}, and in laboratory experiments 0.1 or even 1. No decisive role is accordingly played by this ratio.

2. Phase conversions of moisture are absent from laboratory experiments, that is, there is no precipitation and no release of latent heat or loss of heat to evaporation. In everyday life the nature of weather is associated primarily with precipitation, but all this plays a subordinate role in general circulation processes.

3. In laboratory experiments there is no similarity with natural conditions from the viewpoint of the Reynolds number, that is, the viscosity is many orders of magnitude higher than in reality.

4. The so-called β-effect, that is, change in the Coriolis force with latitude, is absent.

All this demonstrates that not all of the atmospheric parameters play the same role in determining the nature of general atmospheric circulation, and this fact inspires a certain amount of hope. However, it is now time to proceed directly to the su: jects of our study, the planets and their atmosphere.

4. Survey of Astronomic and Atmospheric Parameters of the Solar System Planets

Let us begin with a survey of the necessary astronomic factors (Table 1) determining climate, and accordingly the general circulation: the distance between planets and the Sun R_a (in astronomic units; 1 a.u. = 149,500,000 km), their radiuses r, periods of revolution around the Sun T_p, the length of the day $t = 2\pi/\omega$, the angle of inclination of the axis of rotation to the plane of the ecliptic φ_0, the acceleration of gravity g, and the integral albedo /19 of a planet A. The information on these parameters is taken basically from the book by Moroz (1967). If cases in which ß differing from this information are adopted, specific mention is made of this circumstance whenever it arises. No consideration is given here to Pluto, since for the time being virtually nothing is known about it.

TABLE 1. ASTRONOMICAL PARAMETERS OF THE PLANETS.

Planet	R_a a.u.	r, km	T_p, years	$t=2\pi/\omega$ days	φ_0	g, cm/sec^2	A
Mercury . .	0,39	2 434	0,24	59	28°	388	0,09
Venus . . .	0,72	6 050	0,62	−243	2°	890	0,77±0,07
Earth	1	6 371	1	1	23° 27′	981	0,3
Mars	1,52	3 394	1,88	1,02	24° 57′	370	0,20±0,05
Jupiter . . .	5,2	70 000	11,9	0,41	3° 7′	2500	0,5
Saturn . . .	9,5	60 000	29,5	0,44	26° 44′	950	0,5
Uranus	19,2	25 000	84	0,45	87,5°	900	0,5
Neptune . . .	39,5	24 000	248	0,66	29°	1200	0,5

Commas indicate decimal points.

Of the seven parameters listed here, only the mean distance between a planet and the Sun, R_a, and the period of revolution around the Sun, T_p, are well known (that is, known with "astronomic" accuracy). The accuracy of the remaining parameters ranges from fractions of a percent to several percent.

The radius for Mercury has been adopted on the basis of the radar observations of Ash, Shapiro, and Smith (1967); it is 46 km smaller than the value given by Moroz (1967). It is this which explains the large value of g.

The Venusian radius of 6050 km (with an error of ±5 km) is the one universally adopted at the present time and corresponds to numerous and varied data obtained in radar measurements (see Ash et al., 1968). The acceleration of gravity corresponds to this value and to the mass of the planet, which equals 0.815 of the Earth's mass. The albedo value is based on the data of Irvine (1968).

According to a number of measurements made in recent years (see, for example, Raschke, Bandeen, 1970) the albedo of Earth is 0.3.

The value for the radius of Mars has been borrowed from the data obtained in flight path measurements by the Mariner-9 station (Steinbacher et al., 1972). The albedo value, 0.2, corresponds approximately to the results of averaging over a detailed map of albedo distribution on the surface of the planet prepared by de Vakuler for the numerical experiments of Leovy and Mintz (1966, 1969) relating to simulation of the circulation of the Martian atmosphere.

The mean value of the polar and equatorial values have been adopted for the radiuses of Jupiter and Saturn. Both planets are appreciably flattened because of the speed of their rotation. The values for the radiuses of Uranus /20
and Neptune have been given in accordance with measurements made by Dollfus (1970). The most indefinite values for the large planets are those of the albedo. We everywhere assume A = 0.5, as is recommended by Moroz (1967), although value A may increase to 0.7 or even higher for dense gaseous atmospheres. It is true that allowance for atmospheric absorption lowers the albedo value.

Let us now consider the parameters directly characterizing the atmosphere itself. The chief among these are pressure p, that is, the mass of the unit atmospheric column, $M = p/g$, the chemical composition of the atmosphere, and the molecular weight of the latter, μ. The optically active admixtures are of great importance in assessing radiation, but we do not cite them here because of the great uncertainty regarding their concentrations for all the planets. If μ is known, by use of the equation of state of an ideal gas, one can determine such useful characteristics of the atmosphere as the heat capacity per unit mass at constant pressure, $c_p = \frac{\kappa}{\kappa - 1} \frac{R}{\mu}$, in which $\kappa = c_p/c_v$, $R = 8.314 \times 10^7$ erg/ /(mol·K) is the universal gas constant, and $\gamma_a = g/c_p$ is the adiabatic vertical temperature gradient corresponding to total turbulent vertical mixing of the atmosphere (the entropy being constant with altitude). If there is a mixture of n gases having molar concentrations α_i, then $C_{p0} = \sum_{i=1}^{n} \alpha_i C_{pi}$, and the adiabatic exponent of the mixture κ_0, is in this instance determined from the equation

$$\frac{1}{\kappa_0 - 1} = \sum_{i=1}^{n} \frac{\alpha_i}{\kappa_i - 1}, \tag{4.1}$$

which follows from the formula for the molar heat capacity C_v of a mixture of gases ($C_{vi} = R/\kappa_i - 1$). Generally speaking C_v, and consequently γ_a as well, are functions of pressure and temperature, although fairly weak ones. We shall disregard these relations, assuming the gas to be an ideal one, but for deep atmospheres, such as those of Jupiter, or even Venus, the values of c_p and γ_a vary considerably with depth.

An important characteristic of the thermal conditions of the atmosphere of a planet is represented by the equilibrium temperature of escaping radiation, T_e. If there are no internal sources of energy whatever, the equilibrium temperature is determined from the balance of incident solar energy and the escaping radiation flux

$$\pi r^2 q_A = 4\pi r^2 \sigma T_e^4, \tag{4.2}$$

in which $q_A = q_0(1 - A)$, q_0 is the solar constant for the planet, A its albedo, and $\sigma = 5.67 \cdot 10^{-5}$ erg/(cm$^2 \cdot$ sec$\cdot$K^4) is the Stefan-Boltzmann constant. Hence

$$T_e = \left(\frac{q_A}{4\sigma}\right)^{1/4} = \left(\frac{q}{\sigma}\right)^{1/4}. \tag{4.3}$$

Value T_e serves as a gage of the energy supplied to the atmosphere. It /21
continues to be such even when internal sources of energy are present, as is the case with Jupiter and Saturn (Aumann, Gillespie, Low, 1969). For these planets the values of T_e have been found to be such that the balance condition of (4.2) is not satisfied even when A = 0. It is just for this reason that we select T_e rather than q_0.

If T_e is known, it is possible to estimate the altitude of the homogeneous atmosphere at the level of formation of the escaping radiation:

$$H = \frac{RT_e}{\mu g} \tag{4.4}$$

and the speed of sound at the same level

$$c_e = \left(\frac{\varkappa R T_e}{\mu}\right)^{1/2} = (\varkappa g H)^{1/2}. \tag{4.5}$$

All these values are given in Table 2.

It is to be noted that not all the data contained in the first two columns are in agreement with those given in the book by Moroz (1967). The remaining columns are calculated on the basis of known values μ, $\varkappa$, and the data of Table 1. The pressure at the surface (p_s) of Mercury may be much lower than 1 mb, and the upper limit of the CO_2 content is now estimated at 0.04 mb (see Bergstrahl, Gray, and Smith, 1967). According to the measurements made by Venus-7 (see Marov et al., 1971), M for Venus is near 100 kg/cm^2 (around 90 atmospheres)[3]. The composition of the Venusian atmosphere has been known since the time of the direct measurements made by Vinogradov et al. (1968, 1970), which show that it consists almost entirely of carbon dioxide.

TABLE 2. ATMOSPHERIC PARAMETERS OF PLANETS

Planet	M, g/cm^2	μ	$c_p \cdot 10^{-7}$ erg/g K	$\varkappa$	γ_a K/km	T_e K	H km	c_e, m/sec
Mercury	<0,1	44	0,85	1,28	4,6	500	25	350
Venus	10^5	44	0,85	1,28	10,5	230 + 8	5	240
Earth	10^3	29	1	1,41	9,8	255	7,3	320
Mars	16	44	0.85	1,28	4,4	216	11	230
Jupiter	10^3	2,6	10	1,45	2,5	134 + 4	17	790
Saturn	$2 \cdot 10^3$	2,6	10	1,45	0,93	97 + 4	33	670
Uranus	10^3	2,2	13	1,42	0,76	54	21	540
Neptune	10^3	2,2	13	1.42	1,0	38	11	450

Commas indicate decimal points.

For Mars $p_s = 6 \pm 2$ mb, that is, $M = 16\ g/cm^2$. This i the most probable /22 figure, which follows from the data of Mariner-6 and Mariner-7 (see Rasool, Stewart, 1971). Their data indicate the possible presence of gases other than CO_2, but since the content of such obvious candidates as nitrogen, argon, or neon does not exceed 1% (Barth et al., 1971), for Mars as well we adopt $\mu = 44$.

The atmospheres of Jupiter and Saturn must be very deep. Probably no phase shifts from the gaseous to the liquid or solid state take place at all inside Jupiter, in view of the high temperatures and pressures in its interior (Trubitsyn, 1972; Zharkov, Trubitsyn, Samsonenko, 1971), that is, the matter of the planet is in the supercritical state. For this reason the concept of atmospheric depth loses all meaning in this case.

The pressure (mass) cited in the table for Jupiter and Saturn corresponds to the cloud cover level, at which it is known accurately to a factor of the order of 2. During the 1960s there was observed for Jupiter a tendency toward lowering of the molecular weight estimates from 4 (basically helium) to 2.6 (Moroz, 1967). More recently (for example, see Owen, 1969) there is increasing inclination to adopt the viewpoint that the composition of the large planets is near the distribution of the chemical elements in space. Then $\mu = 2.2$ and $\kappa = 1.42$. We have adopted precisely these values for Uranus and Neptune. For Jupiter and Saturn we have adopted values of T_e indicating internal sources of heat (Aumann, Gillespie, Low, 1969). Unfortunately, lack of precise knowledge

[3]According to the data of measurements made by the Soviet Venus-8 space station (see "Pravda" of 10 September 1972), the pressure on the surface of the planet at the point at which the station landed was $90 \pm 1.5\ kg/cm^2$, that is, the atmospheric column mass equals $99 \pm 1.5\ kg/cm^2$.

of the albedo value for these planets permits only the statement that the intensity of these sources is comparable to the influx of heat from the Sun. No one has measured the temperature for these two last-named planets, and we have adopted them, in accordance with calculations based on formula (4.3), with an albedo value of A = 0.5.

CHAPTER 2. THEORY OF SIMILARITY FOR GENERAL CIRCULATION OF PLANETARY ATMOSPHERES /23

5. Estimates of Certain General Circulation Characteristics Based on Considerations of Energy and Thermodynamics

The first hints that there may be certain simple quantitative mechanisms in operation in planetary atmospheres were received by the author as he became acquainted with the work of Monin (1968). In Section 2 of this book it was demonstrated by three independent methods that the typical duration of synoptic processes is on the order of several days for the Earth's atmosphere (also see Monin, 1969).

As a matter of fact, the power of the solar energy coming towards the Earth's atmosphere equals $1.2 \cdot 10^{17}$ wt, in view of the albedo of the latter. According to the empirical estimates of Palmen (1959), the rate of conversion of potential to kinetic energy $\partial E/\partial t$ is approximately equal to $2 \cdot 10^{15}$ wt. Since the total mass of the Earth's atmosphere is $M_0 = 5.3 \cdot 10^{21}$ g, the conversion rate referred to the unit mass is $\varepsilon = M_0^{-1}\, \partial E/\partial t \approx 4$ cm^2/sec^3. According to the empirical estimates of Borisenkov (1963), Gruza (1965), and Oort (1964), the total kinetic energy of atmospheric motions varies from season to season and amounts to (6 to 9)$\cdot 10^{20}$ j, that is, (6 to 9)$\cdot 10^{27}$ erg. We adopt Oort's estimate of $E = 7.5 \cdot 10^{20}$ j, which was used by Lorenz (1967). Then the typical energy conversion time $\tau = \left(\frac{1}{E} \frac{\partial E}{\partial t} \right) = \frac{7.5 \cdot 10^{20} \text{J}}{2 \cdot 15^{15} \text{W}} = 3.7 \cdot 10^5$ sec $\approx$ 4 days.

The typical degeneration time of the energy of synoptic processes is found to be of the same order, owing to the turbulent viscosity introduced in accordance with Richardson (1926) (see above, Section 2),

$$\tau_U \approx L^2/K \approx \varepsilon^{-1/3} L^{2/3}, \tag{5.1}$$

in which K is the relative diffusion coefficient.

The typical scale of length for synoptic processes is according to Obukhov (1949b) of the order of

$$L_0 = \frac{c}{l} \approx \frac{\sqrt{gH}}{l}. \tag{5.2}$$

In this equation c is the velocity of light, and $l = 2\omega \sin \vartheta$ the Coriolis parameter (ϑ equals latitude); in the temperate latitudes $L_0 \approx 3{,}000$ km. The radius of correlation of meteorological fields of pressure or temperature considered from the statistical viewpoint have the same scale (see, for example, Fortus, 1964). By use of this value of L_0 and the value $\varepsilon = 4$ cm^2/sec^3, we obtain $\tau_U \approx 3 \cdot 10^5$ sec. The Eulerian time scale for synoptic processes, /24

$\tau_1 = L_0/U$, is of the same order, if a characteristic rate of westerly transfer in the atmosphere equalling 10 m/sec is adopted as U.

The first two estimates demonstrate that much useful information may be extracted from knowledge of only one quantity ε, the specific rate of generation or dissipation of kinetic energy in the atmosphere.

The general formula proposed by the author (1968) on the basis of the following considerations may be used to determine ε. The total rate of generation (dissipation) of kinetic energy in the atmosphere represents a certain portion ε of the total flux of solar energy entering the atmosphere, $U_A = \pi r^2 q_0(1 - A)$. Hence if the quantities in question are referred to the unit mass, on the average $\varepsilon = \eta q_A/4M = \eta q/M$. Since all motion in the atmosphere results from uneven heating of the atmosphere, quantity ε, which characterizes the rate of generation of kinetic energy in large-scale general circulation processes, must depend basically on the typical temperature difference in the atmosphere, it being obvious that if $\delta T = 0$, then $\varepsilon = 0$, and consequently $\eta = 0$ as well. Then if we restrict ourselves to the first term of expansion into a series — and it is correct to do so if the temperature departures from the mean are slight — we may write

$$\eta = k\frac{\delta T}{T_1}, \tag{5.3}$$

in which T_1 is the temperature of the most greatly heated parts of the atmosphere and k is a numerical coefficient. As a result,

$$\varepsilon = k\frac{\delta T}{T_1}\frac{q}{M}. \tag{5.4}$$

There is as yet apparently no way of determining k, but its value may be estimated by taking as a basis the empirical materials of the Earth's atmosphere, and, in the case of Mars, the results of the calculations by Mintz and Leovy (1969). As we shall see later on, many of the characteristics in which we are interested depend only slightly on the value of k: rough estimates yield the relation $k^{\cdot 1/3}$, and more precise ones even $k^{1/4}$. Hence it is sufficient to establish merely the order of magnitude of k. It is this circumstance which justifies the broad application of formula (5.4) here to the atmospheres not alone of Earth and Mars but to those of other planets also.

An almost obvious statement of inequality may be written at the very outset for quantity k. As a matter of fact, η performs the role of efficiency ratio of the atmosphere regarded as a heat engine (of the first kind, to use the /25
terminology of V. V. Shuleykin, 1968) in the conversion of solar energy to the kinetic energy of atmospheric motion. Since $\delta T/T_1$ is the efficiency of an ideal heat engine, quantity $k = \eta/\eta_{id}$ may on the analogy with the technical terminology be termed the utilization factor. It is then obvious that

$$k \quad 1, \tag{5.5}$$

since it can hardly be assumed that atmospheres operate as an ideal heat engine.

Successful utilization of formula (5.1) for estimation of the typical times of general circulation processes provides the impetus toward further development of ideas of this kind, for example, for estimations of the velocity

$$U \approx (\varepsilon L)^{1/3}. \tag{5.6}$$

By substituting $\varepsilon = 4\ cm^2/sec^3$ and $L_0 = 3{,}000$ km in (5.6) we obtain $U = 10^3$ cm/sec = 10 m/sec, the typical wind speed on Earth. The coefficient eq[illegible]ng approximately 1.5 (see formula (2.6)) which must be allowed for in this formula) merely improves the result.

Let us estimate the value of k for the Earth's atmosphere, assuming $\delta T \approx 50$ K, $T_1 \approx 300$ K, $q = 2.1 \cdot 10^5\ erg/cm^2 \cdot sec$, and $\varepsilon = 4\ cm^2/sec^3$. Then in accordance with (5.4) $k \approx 0.1$. We shall discuss the value of k in greater detail in Section 12, drawing upon a variety of empirical material, but it is precisely this value which we shall in the main use.

For Mars there are the data of measurement of temperature distributions over the surface (see Moroz, 1967) and a number of temperature calculations in which radiation equilbrium is assumed (Prabhakra, Hogan, 1965; Ohring, Mariano, 1968). Thus, assuming $K = 0.1$ we can estimate ε with (5.4). On the equator of Mars the mean daily temperature $T_1 \approx 250$ K, and on the winter pole $T_2 \approx 150$ K. Then if $p_s = 5$ mb (Leovy, Mintz, 1966), we have $\varepsilon \approx 300\ cm^2/sec^3$. For Mars $L_0 \approx$ 2000 km, since l is almost the same as on Earth and the speed of sound is approximately two-thirds that on Earth (see Table 2). Then $U \approx 40$ m/sec, a value coinciding with the mean speed calculated by Leovy and Mintz (1966). Later we will see that, according to the results obtained by Leovy and Mintz (1969), $k \approx 0.01$ for Mars, and that estimation of the speeds proves not to be such a simple matter, while the agreement between the speed value obtained and the theoretical value is the result of a number of factors as yet not accounted for. Even at $k = 10^{-2}$ we would nevertheless obtain $U \approx 20$ m/sec, that is, again we would not be too far from the actual conditions.

Hence the example of the Earth's and the Martian atmospheres inspires further development of our as yet entirely elementary estimates, which in addition make use of the values of δT and T_1 assigned in advance. At the same time, the last-named values in any general circulation theory that is to any extent complete and self-consistent must be determined by external astronomic factors and the properties of the atmosphere itself, that is, by the data contained in Tables 1 and 2.

Quantity T_1 can be estimated in a quite simple manner: it must be of the same order as equilibrium temperature T_e defined by (4.3). We will write /26

$$T_1 \approx T_e \alpha. \tag{5.7}$$

For the rather shallow atmospheres of Earth and Mars, $\alpha \lesssim 1$ (somewhat less than unity). For deep atmospheres such as that of Venus, α may be estimated if the mass of the atmosphere, the vertical 'iabatic gradient, and the pressure at the level of formation of the escaping radiation are known, these generally being accessible to astronomic observations, or simply if the surface temperature is known, as it is in the case of Venus from the data of radio astronomy or from extrapolation of direct measurements. For Venus $\alpha \approx 230$ K/ /750 K $\approx 1/3$. Since T_1 figures in quantity ε, which further figures in exponent 1/3, the difference between α and unity will for the time being be disregarded.

To determine quantity δT, we use the equation for heat balance in the atmosphere, which after averaging over the altitude in the stationary case may be written in the following simplified form (see Golitsyr, 1970a, b):

$$Mc_p u_i \frac{\partial T}{\partial x_i} \approx \sigma T_e^4. \tag{5.8}$$

According to (5.8) advection of heat in the atmosphere is counterbalanced by radiation into outer space. It is valid if the temperature departures are everywhere small in comparison to the equilibrium value of the temperature, T_e. This takes place when large-scale dynamics play a decisive role in the thermal conditions of the atmosphere. A more detailed quantitative analysis of these conditions will be presented in Section 6 (also see Golitsyn, 1971). The application of equation (5.8) to dark or slightly illuminated regions of the atmosphere requires no explanation, but it is approximately correct in the other cases as well, since the absorption of direct solar radiation in the rather thin atmospheres of Earth and especially of Mars is slight.

Let us consider a slowly rotating planet gradient r of which is the typical space scale of its azonal atmospheric motions. Then

$$U_i \frac{\partial T}{\partial x_i} \approx U \frac{\delta T}{r} \approx (\varepsilon r)^{1/3} \frac{\delta T}{r}. \tag{5.9}$$

From the foregoing and from equation (5.8), we have

$$U \delta T \approx \frac{\sigma T_e^4 r}{M c_p} = \frac{q r}{M c_p}. \tag{5.10}$$

This relation expresses the obvious fact that the mean circulation rate and the temperature difference causing it are closely associated and mutually consistent. It is found that their product may be estimated *a priori*, through the "external parameters" of Tables 1 and 2.

Under the conditions of (4.3) and (5.7) equations (5.4), (5.6), and (5.10) /27
fully define unknown quantities U, δT, and ε. For the present we shall write only the formula for velocity U:

$$U \approx k^{1/4} \frac{\sigma^{1/4}}{c_p^{1/4}} q^{3/4} \frac{r^{1/2}}{M^{1/2}}. \tag{5.11}$$

Thus velocity U and the other unknown characteristics of atmospheric circulation are determined with the precision of empirical constant k merely by means of the external astronomic and astmospheric parameters.

Let us additionally write the expression for the total mean kinetic energy of circulation

$$E = \frac{1}{2} M_0 U^2 = 2\pi r^2 M U^2 \approx 2\pi k^{1/2} \frac{\sigma^{1/2}}{c_p^{1/2}} q^{3/2} r^3. \tag{5.12}$$

That the energy is independent of the mass of the atmosphere is a very surprising fact calling for a fuller understanding that should be gained on the basis of analysis of the atmospheric dynamics equations.

6. Hydrothermodynamic Equations for General Atmospheric Circulation. Similarity Criteria.

The motion of an atmosphere and the conversions of energy in it are governed by the laws of preservation of momentum and mass, the first law of thermodynamics, and the equation of state, for which we will use the equation of state of an ideal gas:

$$\frac{d\mathbf{V}}{dt} = -2[\boldsymbol{\omega}\mathbf{V}] - \frac{1}{\rho}\nabla p + \mathbf{g} + \mathbf{F}, \tag{6.1}$$

$$\frac{d\rho}{dt} = -\rho \operatorname{div} \mathbf{V}, \tag{6.2}$$

$$\frac{dT}{dt} = -(\varkappa - 1) T \operatorname{div} \mathbf{V} + \frac{Q}{c_v}, \tag{6.3}$$

$$p = \frac{R}{\mu} \rho T, \tag{6.4}$$

in which V is the velocity vector, $\vec{\omega}$ is the vector of spin of a planet, ρ is the atmospheric density, g is the acceleration of gravity, F are the mass forces, generally construed as meaning friction, viscous in precise equations or turbulent in meteorological research, $\kappa = c_p/c_v$ is the adiabatic exponent, T is temperature, Q is the heat flux per unit mass, R is the universal gas constant, and μ is the mean molecular weight of the atmosphere.

One of the main difficulties of meteorology lies in adequate description /28
of heat flux Q. In an ideal atmosphere the heat flux may in the first approximation be represented in the form of the difference between the incident solar radiation and the escaping thermal radiation (Lorenz, 1967)

$$Q = qf(\vartheta, \varphi, z, t) - \sigma T^4, \tag{6.5}$$

in which $q = q_0(1 - A)/4$, and function $f(\vartheta, \varphi, z, t)$ allows for the geometric illumination conditions — the dependence on latitude ϑ, longitude φ, and the time of year and day t, and the relationship between absorption and altitude z. On the basis of the results of laboratory experiments (Section 3), we disregard heat fluxes deriving from phase transitions in the atmosphere and from turbulent transfer, which is less than the advective or of the same order as the latter. This approximation is also favored by the results of numerical experiments relating to simulation of the general circulation of the Earth's atmosphere, which make it possible to obtain in the main plausible pictures of circulation with no allowance for these factors.

Formula (4.3) provides a natural scale of temperature based on astronomical factors alone. If the gaseous composition of the atmosphere and the value of the mean molecular weight are known, it is possible to determine the natural velocity scale

$$c = c_e = \left(\frac{\varkappa R T_e}{\mu}\right)^{1/2}, \tag{6.6}$$

in which c_e is the velocity of sound corresponding to temperature T_e. In (6.1)-(6.3) we normalize the velocities to quantity c:

$$U = \frac{V}{c}. \tag{6.7}$$

We indicate the space coordinates in units of the planetary radius r,

$$x_i' = \frac{x_i}{r}, \tag{6.8}$$

and time

$$t' = \frac{r}{c} t = \frac{t}{\tau_e}. \tag{6.9}$$

The density and pressure are normalized to their values at the surface of the planet:

$$p' = \frac{p}{p_s}, \quad \rho' = \frac{\rho}{\rho_s}, \tag{6.10}$$

with

$$\frac{p_s}{\rho_s} = \frac{c_s^2}{\varkappa} \approx \frac{c_e^2}{\varkappa\alpha}, \tag{6.11}$$

in which $\alpha = T_e/T_s < 1$ (T_s is the typical surface temperature; the difference between T_s and T_1 in formula (5.7) is disregarded). We also introduce $\vec{\omega} = \omega n$, /29
$g = gk$, in which n and k are unit vectors. In the dimensionless variables in system (6.1)-(6.4), there appear dimensionless numerical parameters termed similarity criteria because, if the numerical values for two planets are equal, the equations will be identical in both cases, that is, the circulation pictures

will coincide to the precision of the scale factors, and they will be similar in their dimensional variables.

In dimensionless variables equation (6.1) assumes the form

$$\frac{d\mathbf{U}}{dt} = -2\Pi_\omega [\mathbf{n}\mathbf{U}] - \frac{\nabla' p'}{\varkappa\rho'} + \frac{\mathbf{k}}{\varkappa\Pi_g} + \mathbf{F}', \tag{6.12}$$

$$\Pi_\omega = \frac{\omega r}{c}, \tag{6.13}$$

$$\Pi_g = \frac{c^2}{\varkappa r g} = \frac{RT_c}{\mu g r} = \frac{H}{r}. \tag{6.14}$$

In this instance Π_ω is the dimensionless rotational similarity parameter, termed the rotational Mach number. It equals the ratio of the linear velocity of rotation of a planet at the equator to the velocity of sound, c_e. Another meaning of this parameter is the ratio of the radius of a planet to the Obukhov scale, L_0, defined as c_e/ω (see (5.2)).

The second similarity criterion, Π_g, is the ratio of the altitude of a homogeneous atmosphere H to the radius of the planet r. We take into account the well-known fact that large-scale motions are quasistatic, that is, vertical acceleration is small in comparison to g. Equation (6.12) for the vertical velocity vector component is then replaced by the hydrostatic equation

$$\Pi_g \mathbf{k}\nabla' p' = \varkappa\rho', \tag{6.15}$$

while Π_g will not enter into the other equations. Thus its value affects only the vertical pressure distribution and the vertical velocity value.

It can be found by analysis of the equation of discontinuity that the typical vertical velocity value is

$$W \sim \Pi_g U. \tag{6.16}$$

More precise analysis allowing for the spin of a planet, that is, criterion Π_ω, shows that relation (6.16) is the top estimate for W (Kibel', 1957). It is obvious that for all planets $\Pi_g << 1$, that is, the vertical velocities are small in comparison with the horizontal ones; hence the motion are quasihorizontal.

Let us average the energy balance equation (6.3) over altitude. In dimensionless variables it assumes the form

$$\frac{dT'}{dt'} + (\mathbf{U}\nabla')T' = \frac{qr\varkappa}{c_p T_e c M} [f(\theta, \varphi, t) - T'^4], \tag{6.17}$$

in which $T' = T/T_e$. Since $c^2 = (\kappa - 1)\, c_p T_e$, and T_e is defined by (4.3), the dimensionless factor on the right in front of the expression in brackets is written as /30

$$\frac{qr\varkappa}{c_p T_e c M} = \frac{\varkappa}{(\varkappa - 1)^{1/2}} \Pi_M, \tag{6.18}$$

in which

$$\Pi_M = \frac{\sigma^{3/8}}{c_p^{3/2}} q^{5/8} \frac{r}{M}. \tag{6.19}$$

The energy similarity criterion, Π_M, is susceptible of several different interpretations. Let us first consider (6.18). The quantity $Mc_pT_e = I$ in the denominator on the left is the heat content of the unit atmospheric column (accurate to a factor of the order of α), and $r/c = \tau_e$ is the relaxation time of pressure or density disturbances (Obukhov, 1949b) on the global scale, which are propogated at the velocity of sound. Hence $\Pi_M = q\tau_e/I$. On the other hand, as was demonstrated by Gierasch, Goody, and Stone (1970), the quantity

$$\frac{I}{q} = \tau_0 = \frac{c_p M}{\sigma^{1/4} q^{3/4}} \tag{6.20}$$

is a good minimum estimate of the time of establishment of the local radiation equilibrium in the atmosphere for virtually all gases with which we are concerned: water vapor, CO_2, H_2, and so forth. Hence $\Pi_M = \tau_e/\tau_0$ is the ratio of two relaxation times; since the mass of the atmosphere figures in determination of τ_0, it is natural to term it the thermal inertia period of the atmosphere.

In the absence of atmospheric motions, and with the thermal inertia of the atmosphere disregarded, the temperature of the latter would be determined from the condition of local radiation equilibrium $Q = 0$, that is,

$$T_r = \left[\frac{q}{\sigma} f(\vartheta, \varphi, t)\right]^{1/4}. \tag{6.21}$$

As is demonstrated by experience with the Earth's atmosphere, there is a substantial difference between the distribution of the real temperature and the local radiation equilibrium temperature. Hence the normalized heat flux, $Q' = T'^4_r - T'^4$, is usually around unity. Since usually $U \ll c$, it may be assumed that the requirement $\Pi_M \ll 1$ will be a condition of absence of local radiation equilibrium from the atmosphere, and consequently will decide the influence of atmospheric dynamics on formation of the temperature conditions. A more detailed analysis of the situation is to be found in the article referred to by Gierasch, Goody, and Stone (1970). Thus in the stationary case equation (6.17) is written as

$$(U\nabla') T' \approx \Pi_M. \tag{6.22}$$

It can easily be demonstrated that in the dimensional form equation (6.22) /31 coincides with equation (5.8) derived by purely heuristic means.

Hence the basic criteria of similarity constructed on the basis of purely external parameters for the general circulation of planetary atmospheres are the rotational Mach number Π_ω, the ratio of the altitude scale to the radius of the planet Π_g, and energy criterion Π_M. The derivation of the similarity criteria was first presented by the author in 1971. In earlier papers by the author they had been derived simply as dimensionless combinations constructed from the external parameters.

The data of Tables 1 and 2 suffice for construction of the similarity criteria in accordance with (6.13), (6.16), and (6.19), for the planets in the tables.

It is evident from Table 3 that the values of Π_g and Π_M are everywhere small. For this reason, as was stated in Section 2, similarity is to be expected on the basis of them, that is, their precise values (or the precise values of any parameter figuring in them) may be insignificant for determination of certain general circulation characteristics. At the same time, the similarity criterion based on rotation Π_ω is subject to wide variation. It is small for Venus and Mercury, around unity for Earth and Mars, and large for the giant planets. Table 3 suggests that rotation should play a decisive role in determining the atmospheric dynamics and that classification of the planets according to the nature of general circulation in their atmospheres should be made precisely on the basis of the value of the rotational Mach number Π_ω.

TABLE 3. SIMILARITY CRITERIA OF ATMOSPHERIC CIRCULATION FOR PLANETS

Planet	Π_g	Π_ω	Π_M
Mercury	$1\cdot10^{2}$	$8{,}5\cdot10^{3}$	>1
Venus	$8{,}3\cdot10^{4}$	$7{,}6\cdot10^{3}$	$1\cdot10^{5}$
Earth	$1{,}2\cdot10^{3}$	1,43	$1{,}17\cdot10^{3}$
Mars	$3{,}2\cdot10^{3}$	1,05	$3{,}3\cdot10^{2}$
Jupiter	$2{,}4\cdot10^{4}$	15,6	10^{4}
Saturn	$5{,}5\cdot10^{-4}$	14,7	10^{4}
Uranus	$1\cdot10^{3}$	7,5	10^{5}
Neptune	$6\cdot10^{4}$	6	10^{5}

Commas indicate decimal points.

Systematic study of the role of Π_ω has been made by the author in collaboration with Dikiy (1966) (also see Blinova, 1960; Longuet, Higgins, 1968; Dikiy, 1969) for the linearized problem of the natural vibrations of a planetary atmosphere. It has been demonstrated that the eigenfunctions describing the form of the vibrations differ from zero in a certain zone near the equator up to latitudes $\pm\vartheta$, so that $\cos\vartheta \sim \Pi_\omega^{-1/2}$, and the vibrations are rapidly attenuated outside this zone. In physical terms this may be construed as meaning that at large values ω the Coriolis force, which is proportional to $2\omega\sin\vartheta$, extensively suppresses motions in the temperate and high latitudes. This apparently is to be ascribed to the fact that the apparent striated struc- /32 ture on the discs of Jupiter and Saturn extend approximately only to the latitudes of 40-50°. The higher polar regions are uniformly grey in coloration, with no signs of poles whatever. Even the individual spots in the polar regions appear much less frequently than in the equatorial or temperate regions (Peek, 1958; Alexander, 1962).

Before going on to present further material, it is appropriate to establish the relationship between the similarity criteria indicated above and the criteria usually employed in study of the large-scale dynamics of the Earth's atmosphere (see, for example, Monin, 1969), for which characteristic velocity U and typical scale L (for example, the Obukhov scale of (5.2)) are known from observations. There is firstly the Kibel'-Rossby number

$$\mathrm{Ki} \quad \frac{U}{\omega L} \quad \mathrm{Ma}\,\Pi_\omega$$

and secondly the Mach number

$$\mathrm{Ma} = \frac{U}{c} .$$

As we shall see later (see Sections 8, 10), these two similarity criteria are functions of criteria Π_M and Π_ω. The geometric parameter of sphericity, L/r, usually introduced is inversely proportional to Π_ω, and the parameter of quasistaticity, H/L, is proportional to $\Pi_g\Pi_\omega$. Our similarity criteria are so to speak external or primary ones, since they have been constructed entirely from the astronomic and atmospheric parameters of the planets and make no use whatever of observational or *a priori* estimates of characteristics of motion--like velocity, which themselves depend on definition.

The vertical temperature structure of the atmosphere characterized by the Richardson number plays a major role in arriving at a detailed picture of the large-scale dynamics, and in particular the position of the synoptic maximum (see Section 2). This number is a gage of the deviation of the real vertical temperature gradient from the adiabatic. Detailed determination of the vertical temperature structure is one of the difficult tasks of atmospheric physics and requires a larger number of controlling parameters (in particular the absorbing properties of the optically active gases and their vertical distribution, and so forth) than we make use of here. Hence to all appearances one cannot express the Richardson number in the form of a function of only three external similarity criteria.

7. General Similarity Hypotheses for Large-Scale Motions of Planetary Atmospheres

The simple structure of formula (5.12) naturally suggests the idea of attempting to use the considerations of similarity and dimensionality to determine certain mean characteristics of the general circulation of planetary atmospheres. Methods of similarity and dimensionality have been used in study of the most complex hydrodynamic processes, since there are no precise solutions to many problems, usually nonlinear ones, in analytical form, while the numerical solutions are difficult to survey and it is difficult to determine the physical mechanisms from them. Many examples of use of these methods are to be found in the books by Sedov (1971), Birkhoff (1950), Kline (1965), Landau and Lifshits (1954), and so forth. /33

In use of the methods of similarity and dimensionality, the first step is intelligent selection of the dimensional parameters and universal constants which are the most essential for the process being studied. If the problem is formulated in mathematical terms, as in our case, these parameters enter into the equations and the boundary conditions. But generally speaking it is not compulsory to have a precise formulation of the problem; it is sufficient to restrict selection to the controlling parameters from physical considerations, as was done by the author in the first publications on this subject (see Golitsyn, 1970a, b).

Analysis of the physical picture of the processes studied often makes it possible to draw conclusions regarding the insignificance of particular parameters, or rather regarding the insignificance of their exact values in determination of a number of quantities sought. Such conclusions may sometimes be made at once, and sometimes after analysis of the values of the dimensionless criteria of the problem and clarification of their physical significance. The conclusion regarding insignificance of particular dimensional parameters is of course a physical hypothesis rigorous proof of which usually cannot be provided. The justification of such an hypothesis is ultimately represented by agreement between the results obtained by its use and experimental data.

The smaller is the number of dimensional parameters, the larger is the number of rigid functional relationships established between the unknown quantities and these parameters. If there are n parameters, k of which possess independent dimensions, by means of the so-called π theorem (for proof see, for example, the books of Sedov and Birkhoff cited above) it is possible to construct n - k dimensionless complexes on which our unknown quantities will depend. If n = k, that is, if all the controlling parameters have independent dimensions, the unknown quantities accurate to the dimensionless constant determined in theory or empirically are algebraic monomials of these parameters. Experience shows that this dimensionless constant is usually on the order of unity.

The system of equations describing the general circulation is represented by equations (6.1)-(6.4). The controlling dimensional parameters in it are the angular velocity of spin ω[sec^{-1}], the acceleration of gravity g[cm/sec^2],

the dimensionless adiabatic exponent $\kappa = c_p/c_v$, and the heat capacity per unit mass at constant pressure $c_p[\mathrm{cm}^2/\mathrm{sec}^2 \cdot \mathrm{K}]$. If system (6.1)-(6.4) is written in spherical coordinates, a radius vector appears up to the center of the planet. The unit of measurement of this radius vector is the apparent radius of the planet $r[\mathrm{cm}]$. The heat flux expression (6.5) also includes the solar constant for the planet with allowance made for the albedo, $q[\mathrm{g/sec}^3]$, and the Stefan-Boltzmann constant $\sigma[\mathrm{g/sec}^3 \cdot \mathrm{K}^4]$. Lastly energy equation (6.17) also includes the unit column mass $M[\mathrm{g/cm}^2]$. The physical hypothesis that we adopt may be /34
regarded as disregard of other energy flux mechanisms introducing new dimensional constants. Arguments justifying this hypothesis have been presented earlier.

In addition, in equation (6.1) the expression for F also includes kinematic viscosity ν. However, all experience gained in study of the hydrodynamics of large systems indicates that, at Reynolds numbers Re >> 1, the exact value of this coefficient is insignificant, although it is precisely the molecular viscosity that ultimately ensures dissipation of the kinetic energy to heat. The value of μ determines only the scales at which the dissipation takes place (see Section 2), and not the magnitude of the latter. For Earth Re $\sim 10^{12}$, and for Mars Re $\sim 10^{10}$. It is to be noted that in laboratory and numerical experiments Re $\sim 10^3$, they nevertheless reproduce the basic features of atmospheric circulation well. As regards turbulent friction, we will subsequently make use of it, not as an external parameter to be assigned but as a quantity to be determined. Similarly, the circulations are similar on the basis of the Peclet number Pe = UL/ξ as well, where ξ is the coefficient of molecular thermal conductivity.

To recapitulate, we have 6 dimensional parameters: ω, g, c_p, q, r, and M, one universal dimensional constant σ, and one dimensionless constant κ. The last-named constant, as is shown by Table 2, undergoes very little change from planet to planet; for this reason we shall exclude it from consideration from this point onward. In theory, however, all the universal functions of dimensionless parameters and the constants obtained below should also depend on κ, a circumstance which may introduce additional dispersion into their empirically determined values.

Of the seven dimensional quantities indicated, only four possess independent dimensions, since four primary dimensions (length, time, mass, and temperature) figure in their determination. Hence, in accordance with the ρ theorem of the theory of dimensionality, it is possible to construct three independent dimensionless complexes. We select as the latter the similarity criteria already introduced, Π_g, Π_ω, and Π_M, the values of which for the planets are given in Table 3.

Since Π_g is small for all the planets, the hypothesis may be advanced that its precise value in the first approximation is insignificant in determination of the mean characteristics of general circulation. The only parameter figuring exclusively in Π_g but not in Π_ω and Π_M is the acceleration of gravity g. Hence the hypothesis regarding the similarity of certain mean circulation characteristics relative to Π_g when $\Pi_g \ll 1$ may be reformulated as the hypothesis of similarity relative to the exact value of g as soon as it is "sufficiently large". In Section 6 we saw that a small value of Π_g denotes small values of the vertical velocities depending on Π_g or g in comparison to the horizontal ones, which do not depend on Π_g. Thus, for example, the total kinetic energy of circulation need not depend on g in the first approximation. On this basis we subsequently exclude g from the number of controlling parameters, assuming it to be sufficiently large. We may note that, if the acceleration of gravity were not to exist at all in the system ($g = 0$), no convection would occur, and differential heating would not set the atmosphere in motion. The situation here is the same as in analysis of the role of kinematic viscosity: if it were not to exist at all, kinetic energy would constantly be increased. The small but finite values of viscosity limit this growth, but if the Reynolds number is large, the exact value of the viscosity is insignificant. /35

The following similarity criteria based on rotation Π_ω are understandable in physical terms. When $\Pi_\omega \ll 1$, the role of the Coriolis force is small, and ω may then be excluded from consideration. But with very small values Π_ω this parameter begins to play an unexpected new role, which will be discussed in Section 10.

The fact that for all the planets (except Mercury) $\Pi_M \ll 1$ makes it possible to advance the hypothesis of similarity relative to this similarity criterion as well, as soon as the thermal inertia of the atmosphere is sufficiently large. This means that one of the dimensional parameters entering into Π_M is insignificant, or rather the exact value of this parameter is insignificant. Without affecting the values of similarity parameters Π_g and Π_ω it is possible to exclude the mass of the unit atmospheric column M, it being assumed to be sufficiently large, so that $\Pi_M \ll 1$. This hypothesis, like the others, can be verified only by experiment.

8. Slowly Rotating Planets with Dense Atmospheres

When $\Pi_g \ll 1$, $\Pi_\omega \ll 1$, and $\Pi_M \ll 1$ the exact values of g, ω, and M are insignificant, in accordance with the similarity hypotheses. Only four dimensional quantities remain: q, c_p, r, and σ. From them it is possible to construct a unique combination having the dimensions of temperature (which will be the effective equilibrium temperature, T_e (see (4.3.), velocity, c_e (see (6.6)), time, $\tau_e = r/c_e$, and energy. The last-named dimension is of the form

$$E = 2\pi B \frac{\sigma^{1/8}}{c_p^{1/2}} q^{7/8} r^3, \qquad (8.1)$$

in which B is a certain numerical coefficient, and factor 2π has been introduced for the sake of abbreviating certain subsequent formulas.

Even if we were not familiar with formula (5.12), we would be forced to conclude that it is the total kinetic energy of circulation, since the total internal energy of the atmosphere is proportional to the mass of the latter. However, the decisive aspect is found to be comparison with observations and numerical experiments.

Only Venus satisfies the condition of smallness for all three criteria, but it may be assumed that, even when $\Pi_\omega \sim 1$, that is, for Earth and Mars, formula (8.1) will not be too far from reality. In Table 4 the observed or calculated values for E are presented for Venus, Earth, and two models of /36
Mars with two values of pressure at the surface, p_s = 5 and 7.5 mb (see the references in Section 1), and theoretical values of E/B based on (8.1) for various planets.

TABLE 4.

Planet	$E_{exp} \cdot 10^{-26}$ erg	$E/B \cdot 10^{-26}$ erg
Venus	500	50
Earth	60–90	77
Mars	1,2–1,6; 5 mb 1,4–1,7; 7,5 mb	7,0

Commas indicate decimal points.

It is to be seen that there is also an agreement between the theoretical and the experimental values for Earth, and to a lesser extent for Mars. Calculations for the two models of the atmosphere of Mars with different masses even confirms the virtual independence of the total kinetic energy from the mass of the atmosphere. The agreement for Venus is not as good, although the theoretical and the experimental results differ only by a factor of 3 on the basis of the velocity. A detailed discussion of all the planets will be given in Chapter 3, along with a special explanation of the reasons for the greater differences for Mars (see Section 13) and Venus (see Section 14).

With (4.3), (6.6), and (6.9), taken into account formula (8.1) may be written as

$$E = 2B(\varkappa - 1)^{1/4} q\pi r^2 \frac{r}{c_e} = \frac{1}{2} B(\varkappa - 1)^{1/4} Q\tau_e, \qquad (8.2)$$

that is, the total kinetic energy of the atmospheric motions is equal, correct within unity, to the total intensity of solar radiation supplied to the atmosphere of the planet multiplied by relaxation time τ_e.

Comparison of (5.12) with (8.1) yields

$$B \approx k^{1/2}. \qquad (8.3)$$

If the mass of the atmosphere is known, it is possible to determine the mean velocity of atmospheric motions

$$U=\left(\frac{E}{2\pi r^{3}M}\right)^{1/3}=B^{1/4}\frac{a^{1/16}}{c_p^{1/4}}q^{7/16}\frac{r^{1/4}}{M^{1/4}}=\left(\frac{B\Pi_M}{\varkappa-1}\right)^{1/4}c, \tag{8.4}$$

from which it follows that similarity criterion Π_M equals the square of the Mach number, within a factor of the order of unity. As will be demonstrated in Section 10, it is true that this interpretation is valid only for small values Π_ω.

Yet another useful consequence of formula (8.1) is the independence of the mean kinetic energy of the unit volume, or, what amounts to the same thing, the mean wind pressure $1/2\rho U^2$, from the atmospheric pressure.

The results obtained with the aid of considerations of similarity and dimensionality are restricted by formula (8.1) and its direct consequences. Two courses may be followed in order to determine the other characteristics of circulation. In the case of small values Π_ω, when the influence of the Coriolis force is slight, transfer of heat from more greatly heated to cooler areas is effected by direct atmospheric motions. Then the approximate heat transfer equation in the form of (5.10) is valid; if U is known, it is possible to find from it the typical temperature difference δT, as well as dissipation rate ε, the efficiency ratio of the atmosphere η, and then the characteristic circulation lifetime /37

$$\tau_U=\frac{E}{4\pi r^2 M\varepsilon}=\frac{E}{\epsilon}, \tag{8.5}$$

which proves to equal $r/2U$. The circulation lifetime is found to be comparable to the typical circulation space scale divided by the mean velocity. A situation such as this is characteristic of large-scale turbulent mixing.

The other course lies in postulating the circulation lifetime in the form of the equation

$$\tau_U=\frac{\beta}{2}\frac{r}{U}, \tag{8.6}$$

in which parameter β may be termed the degree of organization or degree of ordering of the flux[4]. When $\beta \sim 1$, something which, as we have just seen, occurs

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[4]The physical import of degree of ordering β may be clarified by the following consideration. If the heat fluxes are excluded, the potential energy is prevented from being converted into kinetic energy, the attenuation of the kinetic energy of circulation per unit mass may be described by the equation $dU^2/2dt = -\varepsilon$. Dissipation rate ε has the dimension of the cube of velocity divided by the length scale. Let $\varepsilon = U^3/\beta_1 r$ in which β_1 is a coefficient making total allowance for whatever velocity (and length) scales determine dissipation. The solution of equation $dU^2/dt = -2U^3/\beta_1 r$ with the initial condition $U = U_0$ at $t = 0$ is of the form $1/U - 1/U_0 = t/\beta_1 r$. Hence the energy attenuation time by n times equals $t_n = \beta_1 r/U_0(\sqrt{n} - 1)$, that is, $\xi = 2\beta_1(\sqrt{n} - 1)$. Hence the larger the value of β, the slower are the velocities at which dissipation occurs, that is, the more greatly ordered does the large-scale flux become.

when $\Pi_\omega \ll 1$, large-scale mixing of the turbulent type occurs, and at large values Π_ω the flux acquires an organized structure and may reach high velocities.

This definition of τ_U may also be adopted at arbitrary values Π_ω, it being found that β is an essential function of Π_ω. Use of energy transfer equation (5.8) at arbitrary values Π_o becomes ineffective, since in geostrophic motions the temperature gradient is for the most part perpendicular to the velocity vector. The introduction of β makes it possible to dispense with this equation. If the velocity and the dissipation are known, β may be found from the formula

$$\beta = \frac{2EU}{\epsilon r}, \tag{8.7}$$

which follows from (8.5) and (8.6). Effective determination of the value of β for a number of general circulation models will be made in Section 11 (also see Golitsyn, Zilitinkevich, 1972). /38

With τ_U known, we further determine ϵ and ε, and then η and δT, in accordance with (8.5). The structure of the formulas is the same for all the quantities indicated:

$$\frac{\tau_r}{\tau_U} \approx \frac{\beta\epsilon}{k^{1/4}Q} = \frac{\beta\eta}{k^{1/4}} \approx \frac{\beta k^{1/4}\delta T}{T_r} \approx k^{1/4}\Pi_M^{1/4}. \tag{8.8}$$

The reader can readily derive for himself precise expressions for these quantities by means of the external paramete and constant B. It is believed that he will be pleased with the excellent exponents in which the parameters in question will figure, especially energy flux q.

If the restriction of (5.5) $k \ll 1$, is remembered, it follows from formulas (8.4) and (8.8) that at the assigned external parameter value for the planet, there are theoretical top limits for mean velocity U, efficiency ratio η, dissipation rate ϵ, or ε, and bottom value for circulation lifetime τ_U and characteristic temperature difference δT.

There is yet another obvious inequality for temperature difference δT, this time a maximum one:

$$\delta T < \gamma_i T_e, \tag{8.9}$$

in which $\gamma_i = 1 - T_c/T_e$, and T_c is the condensation temperature of the gas saturating the atmosphere. The latent heat released on condensation prevents further drop in temperature. This inequality is of interest primarily for Mars. For the latter $T_c \approx 150$ K, and $T_e \approx 220$ K; hence $\gamma_i \approx 1/3$. For Earth $T_e = 90$ K, the condensation temperature of oxygen; hence $\gamma_i \approx 2/3$. It is useful to keep this inequality in mind at values Π_M which are not too small, when use of the corresponding expression for δT may violate inequality (8.9).

9. Slowly Rotating Planets with Rarefied Atmospheres

For finite values of criterion Π_M factor B in formula (8.1) ceases to be constant and must be regarded as a function of Π_M, that is,

$$B = B_0 f(\Pi_M),\ f(0) = 1. \tag{9.1}$$

Function $f(\Pi_M)$ should be a decreasing function of Π_M, since otherwise statement of inequality (8.9) would be valid.

We shall retain approximate equation $B \approx k^{1/2}$ for quantity B_0 as well. Bearing in mind that $\delta T \approx B^{3/2}(k\beta)^{-1}\Pi_M^{1/2}\, T_e$, from (8.9) we then obtain

$$f(\Pi_M) < (\gamma_1\beta)^{2/3} k^{1/6} \Pi_M^{-1/3}. \tag{9.2}$$

As a matter of fact, $f(\Pi_M)$ should begin to decrease fairly rapidly even at small values Π_M. This can be demonstrated by calling attention to a certain feature of the formulas we have derived. According to (8.8) $\delta T \sim q^{9/16}$, while $T_e \sim q^{1/4}$. The characteristic temperature of the cold part of the atmosphere, /39

$$T_2 = T_1 - \delta T = \frac{T_e}{\alpha}\left(1 - \frac{\delta T}{T_1}\right) \approx \frac{T_e}{\alpha}\left(1 - \frac{B^{3/2}\alpha}{k\beta}\Pi_M^{1/2}\right)$$

increases with increase in T_e only up to definite values $\bar{\Pi}_M$ if function $f(\Pi_M)$ is not taken into account. Since $\Pi_M^{1/2} \sim q^{5/16} \sim T_e^{5/4}$, taking (3.3) into account, we obtain from the condition $dT_2/dT_e > 0$

$$\bar{\Pi}_M < \frac{16}{81}\frac{k^2\beta^2}{B_0^3} \approx \frac{16}{81} k^{1/2}\beta^2. \tag{9.3}$$

If $k \approx 0.1$, $\beta \approx 1$, then $\bar{\Pi}_M < 0.06$; if $k \approx 10^{-2}$, a circumstance we shall later see to be characteristic of Mars, then $\bar{\Pi}_M < 0.02$.

Thus if no allowance is made for function $f(\Pi_M)$, an unnatural picture may be obtained even at small values Π_M: flow of heat toward the planetary atmosphere may lead to drop in temperature, say at the poles, and vice versa. In order for this not to take place, function $f(\Pi_M)$ must begin to decrease fairly rapidly with increase in Π_M, at any rate earlier than Π_M reaches several hundreds.

Mercury may prove to be a planet for the atmosphere of which $\Pi_M \gg 1$. Large values Π_M correspond to low atmospheric density; hence it is necessary to make certain that the Reynolds number remains fairly large and does not thereby violate the similarity relative to kinematic viscosity ν. If $Re = cr/\nu$ is selected, the condition $Re \gg 1$ will be equivalent to the requirement of a short mean free path l in comparison to the radius of the planet, inasmuch as $\nu \approx cl$.

In the work of Gierasch, Goody and Stone (1970) referred to earlier, it was demonstrated that the case $\Pi_M >> 1$ corresponds to the presence of local radiation equilibrium in the atmosphere determining the horizontal temperature distribution, to which the atmospheric dynamics is adjusted. In addition, a rarefied atmosphere transfers too little heat to play a substantial role in establishment of temperature distribution.

On the basis of general considerations regarding the possibility of existence of circulation similarity conditions at $\Pi_M >> 1$ and the requirement of fairly rapid decrease in function $f(\Pi_M)$ with increase in Π_M, the author (see Golitcyn, 1970b) proposed the asymptotic function $f(\Pi_M) \sim \Pi_M^{-7/5}$ when $\Pi_M >> 1$. This function is obtained on the assumption of similarity of the general formula for kinetic energy, (8.1), to the value of B in accordance with (9.1), relative to energy flux q, q being assumed to be fairly large, so that the value of $\Pi_M \sim q^{5/8}$ is large in comparison to unity. At the present time there are no data whatever arguing for or against this hypothesis, but from /40
the physical standpoint it remains possible for the time being. Such data could be obtained only by numerical simulation of circulation on a hypothetical planet with $\Pi_M >> 1$, but $\Pi_M << 1$.

10. Similarity of Circulation with Rotation Taken Into Account

Let us consider planets having a finite rotational similarity criterion Π_ω but a small energy criterion Π_M. We again use the basic formula for the total kinetic energy of circulation, (8.1), which does not depend on Π_M, but now constant B will be assumed to be a function of Π_ω, that is,

$$B = B_0 f_1(\Pi_\omega), \; f_1(\Pi_\omega) = 1 \text{ for } \Pi_\omega < 1, \; B_0 \approx k^{*}. \qquad (10.1)$$

It follows from the general considerations regarding the stabilizing role of rotation that function $f_1(\Pi_\omega)$ should increase with increase in Π_ω, since at high speeds of rotation the development of disturbances in the flow is retarded and the scales of these disturbances are reduced; at the assigned energy (and dissipation) flux the mean flow becomes more ordered and may reach high velocities.

In the case of arbitrary values of Π_ω the form of function $f_1(\Pi_\omega)$ cannot be determined, and explicit formulas are not obtained for the characteristics of atmospheric circulation, as they were in Section 8, but the relation $U \sim \tau_U^{-1} \sim \in \sim \eta \sim \delta T \sim M^{-1/2}$, $\varepsilon \sim M^{3/2}$ is retained here.

With $\Pi_\omega < 1$ we shall confine ourselves to the first terms of development of function $f_1(\Pi_\omega)$ into a Taylor series:

$$f_1(\Pi_\omega) \approx 1 + a\Pi_\omega^2, \; a > 0. \tag{10.2}$$

The term commensurable with Π_ω must be small or altogether absent, since change in the sign of ω, that is, in the direction of rotation, should not change the circulation characteristics (such terms may exist whenever the period of rotation is not small in comparison to the annual period or when large-scale asymmetric orography is present). As we shall see later, in analysis of observational data for Earth and numerical experiments for Mars, function $f_1(\Pi_\omega)$ may in reality be fairly well approximated by relation (10.2) with $a \approx 1$. The model estimates of Section 11 demonstrate that function $f_1(\Pi_\omega) \sim \Pi_\omega^2$ may be satisfied also at larger values Π_ω. We may note that here as well $\beta \sim \Pi_\omega^2$.

However, an unpleasant situation of a very special kind may be anticipated at very small values of Π_ω. The trouble is that small values Π_ω correspond to solar days of great length, and the dark side of the planet may cool off more than the poles during the long night, and then the circulation picture is altered: circulation will be directed mostly from the light side to the dark and not from the equator toward the poles. The scale of circulation will in this instance increase to πr; other conditions being equal this should lead to increase in the velocities, or in our terms in function $f_1(\Pi_\omega)$. The criterion for existence of a particular set of circulation conditions can be /41 obtained by comparing δT referred to the distance between the pole and the equator (see the last formula of (8.8)), with the change in temperature during the night resulting from cooling of the atmosphere. If the length of the night τ_d is much shorter than the thermal relaxation time of the atmosphere, τ_0 (see 6.20)), then

$$\delta T_d \approx \frac{\tau_d}{\tau_0} T_e \approx \pi \frac{\Pi_M}{\Pi_\omega} T_e. \tag{10.3}$$

Comparing this estimate with δT obtained from (8.8), we obtain

$$G = \frac{\delta T_d}{\delta T} \approx \pi \frac{\Pi_M^{1/2}}{\Pi_\omega}. \tag{10.4}$$

In $G \ll 1$, a major role in circulation is played by the cold poles, and in $G \gg 1$, the decisive role is played by the temperature difference between the daytime and nighttime hemispheres.

For very large values Π_ω the author (see Golitsyn, 1970b), on the basis of the possibility of existence in this instance of similarity conditions and the requirement of $\partial f_1/\partial\Pi_\omega > 0$, that is, increase in circulation energy with acceleration of rotation, proposed the formula

$$f_1(\Pi_\omega) \sim \Pi_\omega^7, \tag{10.5}$$

but because of the small amount of observational material available it is for the time being not possible to confirm (or reject) this formula, and with this we shall conclude our discussion of rotating planets.

11. Other Estimates of Global Circulation Characteristics Based on Various Hypotheses Regarding the Nature of Dissipation

On the basis of similarity hypotheses formula (8.1) was established in Section 8 for the total kinetic energy of circulation. Employment of the thermodynamic expression for the mean specific rate of dissipation of kinetic energy, ε — formula (5.4) together with the hypothesis regarding the nature of mixing expressed by formula (8.6) — permits estimation of ε and typical temperature difference δT in accordance with (8.5). These considerations, supplemented by simplified heat transfer equation (5.8) or (6.22), make it possible to obtain all the results even without use of similarity hypotheses as was done in Section 5.

In Section 5 and subsequently it was assumed that the basic dissipation of kinetic energy takes place in the free atmosphere, and that large-scale mixing obeys the Richardson-Obukhov law. There are no grounds for considering these assumptions to be everywhere valid: for the Earth's atmosphere, for example, they may be regarded only as an approximation. We know that in the Earth's atmosphere around 1/2 or more of the total dissipation is accounted for by the boundary layer of the atmosphere (Zilitinkevich, 1970). Atmospheric /42 turbulence of the largest scale obeys the "k^{-3} law" (see Section 2), and although this does not contradict the statement that large-scale relative diffusion obeys the Richardson law, the situation nevertheless still remains vague and completely universal applicability of formulas like (5.6) doubtful.

Thus it is of interest to consider a case in which the bulk of dissipation is accounted for by the boundary layer of the planetary atmosphere. In this instance the laws governing resistance for a boundary layer are replaced by hypotheses regarding the nature of mixing, and employment of the thermodynamic expression for ε and the simplified heat transfer equation obtained under very general conditions makes it possible to complete the problem. The nature of the boundary later depends on the speed of rotation of the planet, and so here as well we shall classify these layers as a function of the value of the rotational Mach number, Π_ω. In restricting ourselves to allowance for dissipation in the boundary layer alone, we of course will be dealing with rather idealized models, but a formulation of the problem such as this introduces a certain amount of clarity into understanding of the basic mechanisms and the nature of the quantitative relationships of the general circulation of planetary atmospheres.

In the second part of the section consideration is also given to various hypotheses, again differentiated as a function of the value of Π_ω, regarding the nature of mixing and heat transfer, when the bulk of dissipation takes place in a free atmosphere, that is, a fairly deep one such as the atmospheres of the large planets. This section follows the article by Golitsyn and Zilitinkevich (1972) in content.

A. Dissipation in the Boundary Layer

A.1. Slow Rotation

When $\Pi_\omega \ll 1$ the rotation is insignificant and circulation of the Hadleyan type is to be anticipated, when heat advection takes place at the velocity of the basic flow U in the direction opposite temperature gradient $\delta T/r$. Then in place of (5.8) we may use simplified equation (5.10)

$$c_p M U \delta T \simeq qr. \tag{11.1}$$

The mean rate of dissipation of kinetic energy in the atmosphere per unit of mass of the atmosphere equals, in accordance with (5.4),

$$\varepsilon = k \frac{\delta T}{T_1} \frac{q}{M}. \tag{11.2}$$

The local rate of dissipation of the energy of mean motion due to friction in the boundary layer is expressed by the formula (see Zilitinkevich, 1970)

$$\varepsilon_z = -\left(\frac{\vec{u}}{\rho} \frac{\partial \vec{\tau}}{\partial z}\right), \tag{11.3}$$

in which u and $\vec{\tau}$ are the horizontal wind speed vector and the vertical momentum flow, ρ is the atmospheric density, and z is the vertical coordinate. If the bulk of dissipation does in fact take place in the boundary layer, the dissipation value which is the mean for the entire depth of the atmosphere is expressed in the form /43

$$\varepsilon = \frac{1}{M}\int_0^\infty \varepsilon_z \rho\, dz = \frac{1}{M}\int_0^\infty \left(\vec{\tau} \frac{\partial \vec{u}}{\partial z}\right) dz. \tag{11.4}$$

Integration has been carried out here by parts, with allowance made for the boundary conditions: $\vec{u} = 0$ when $z = 0$ and $\tau \to 0$ when $z \to \infty$. The basic contribution to the righthand member of (11.4) is made by the surface boundary layer integral $0 < z < h$, within the limits of which momentum flow τ is approximately constant in altitude (above this layer changes in speed with altitude are not as significant, and $\vec{\tau}$ decreases sharply). Designating the modulus of velocity at altitude h by U, and the gas density and momentum flow modulus at the surface of the planet respectively by ρ_0 and τ_0, and utilizing the coefficient of resistance

$$c_0 = \frac{\tau_0}{\rho_0 U^2}, \tag{11.5}$$

we find

$$M\varepsilon \simeq c_0 \rho_0 U^3. \tag{11.6}$$

Now setting the wind speed at altitude h to be identically equal to the typical speed value in the atmosphere, from (11.1), (11.2), and (11.6), taking into account the formula $\rho_0 = gM/R'T_s$ (T_s being the characteristic surface temperature of the planet and $R' = R/\mu$ the specific gas constant) and $c_p/R' = \kappa/\kappa - 1$), we obtain

$$U^4 \approx \frac{\varkappa - 1}{\varkappa}\,\frac{k}{c_0}\,\frac{rq^2}{gM^2}. \qquad (11.7)$$

To be more precise, the righthand member should also include the factor

$$\frac{T_e}{T_s} \approx \frac{T_e}{T_1} = \alpha.$$

Parameter α has already been considered in Section 5.8. For atmospheres which are not very dense this factor is only slightly smaller than unity and it need not be written out.

In accordance with (11.7) we have the following expression for the total kinetic energy of circulation

$$E = 2\pi r^2 MU^2 \approx 2\pi\left(\frac{\varkappa - 1}{\varkappa}\right)^{1/2}\left(\frac{k}{c_0}\right)^{1/2}\frac{qr^{5/2}}{g^{1/2}}. \qquad (11.8)$$

As was the case earlier, the expression here for energy again does not include the mass of the atmosphere, but unlike (5.12) or (8.1) expression (11.8) now depends on g and does not depend on the heat capacity of the gas. We designate by E_0 the energy defined by (8.1). /44

Comparing (11.8) and (8.1), (8.2) and (6.14) being taken into account, we find

$$\frac{E}{E_0} \approx \left(\frac{\varkappa - 1}{\varkappa}\right)^{\cdot}\frac{\Pi_g^{1/4}}{c_0^{1/4}}. \qquad (11.9)$$

Coefficient of resistance c_0 depends on the vertical stratification of the atmosphere, but not too heavily, varying by no more than one-half order of magnitude even in the case of extreme departures of the stratification from the neutral (Zilitinkevich, 1970). In the case of neutral stratification $c_0 = \kappa^2/ln^2(h/z_0)$, in which $\kappa \approx 0.4$ is the Karman constant and z_0 the roughness altitude. For rough estimates it may be assumed that $c_0 \approx 10^{-3}$. According to Table 3, for all planets $\Pi_g \approx 10^{-3}$; hence $E/E_0 \approx 1$, that is, the two formulas (8.1) and (11.8) will yield virtually identical values of the total kinetic energy of circulation.

The approach presented makes it possible to find parameter β, the degree of ordering of large-scale motions defined by formula (8.7). It is found to equal

$$\beta \approx \frac{\Pi_g}{c_0 \alpha}. \qquad (11.10)$$

If it is considered that $\Pi_g \approx c_0$, $\beta \approx \alpha^{-1}$, that is, β may be of the order of unity or slightly larger. In Section 8 this followed from the hypotheses regarding the nature of mixing and from the heat transfer equation.

Typical temperature difference δT can be determined without difficulty by means of equations (11.1) and (11.7). When we compare this difference with the corresponding expression derived from formula (8.8), which we designate as δT_0, we find that

$$\frac{\delta T}{\delta T_0} \approx \beta \left(\frac{c_0}{\Pi_g} \right)^{1/4}, \tag{11.11}$$

a value also near unity being obtained.

Thus both approaches yield virtually the same results when $\Pi_g \approx 10^{-3}$. In order to clarify the range of applicability of each of them it would be of interest to simulate circulation on hypothetical planets with values of Π_g differing greatly from 10^{-3}. It is natural to assume, and this is confirmed by simple calculations (see Golitsyn, Zilitenkevich, 1972), that when $\Pi_g \ll 10^{-3}$ the relative role played by the boundary layer decreases, and then formula (8.1) is valid, while the opposite situation prevails when $\Pi_g \gg 10^{-3}$, and formula (11.8) is found to correspond more closely with the actual conditions.

A.2. Rapid Rotation

If the value of Π_ω is not small, the term including the Coriolis acceleration becomes essential in the motion equations. The motions in the free atmosphere will be near the geostrophic ones, and the planetary boundary layer will be of the Ekman type. Systematic motions in the atmosphere acquire a near nature zonal. In such zonal flow the meridional flows may arise as a result of turning of the wind with altitude in the Ekman boundary layer. In the following estimates of a particular point we will assume that it is precisely this velocity component that effects the basic heat transfer along the meridian, and will discuss the consequences of such an assumption. /45

We will make use of the Ekman equations (see Zilitenkevich, 1970):

$$2\omega_z \rho v + \frac{\partial \tau_x}{\partial z} = 0, \quad -2\omega_z \rho (u - U) + \frac{\partial \tau_y}{\partial z} = 0, \tag{11.2}$$

in which u and v are the latitudinal and meridional velocity components, U is the geostrophic speed or typical speed of the wind in the free atmosphere, motion in which is assumed to be purely zonal, τ_x and τ_y are the latitudinal and meridional components of the momentum flow, and $\omega_z = \omega \sin\vartheta$ is the projection of the vector of angular rotational velocity of the planet in the vertical direction. The first of equations (11.2) yields the following estimate of the mean meridional speed for the atmosphere:

$$V \approx \frac{1}{M} \int_0^\infty v \rho dz = \frac{\tau_{x0}}{2M\omega_z} \approx \frac{\tau_{x0}}{M\omega}. \tag{11.13}$$

The component of frictional stress at the surface, τ_{x0}, figuring in this equation may be estimated with the formula

$$\tau_{x0} = c_x \rho_0 U^2. \tag{11.14}$$

In this formula c_x is a coefficient of resistance of the form $c_x = c_g^2 \cos\gamma$, in which c_g is the geostrophic coefficient of friction and γ the wind shift angle in the boundary layer. Let us assume that on the average the stratification in the boundary layer is near the neutral. Then c_x will depend only on the value of the Rossby number, $Ro = U/\omega z_0$. In accordance with the resistance law for an Ekman boundary layer (Zilitenkevich, 1970), this dependence is found to be rather slight, since on change in Ro c_g^2 and $\cos\gamma$ are opposite in behavior. For rough estimates c_x may be assumed to be constant and to be approximately equal to 10^{-3}. Using the equation of state and formulas (6.14), (11.13), and (11.14), we obtain

$$V \approx \frac{c_x \alpha U^2}{r\omega \Pi_g}. \tag{11.15}$$

Heat transfer equation (6.22) or (5.8) may in the case under consideration be written in the form

$$V\frac{\delta T}{r} \approx \frac{q}{Mc_p}. \tag{11.16}$$

Expression of dissipation by means of the wind speed remains to be done in order to complete the system. For this purpose we substitute in (11.3) $\partial\tau_x/\partial z$ and $\partial\tau_y/\partial z$ from Ekman equations (11.12). After simple conversions we obtain the precise formula /46

$$\rho\varepsilon_A = U\frac{\partial\tau_x}{\partial z},$$

from which it further follows that

$$\varepsilon \approx \frac{1}{M}\int_0^\infty \varepsilon_A \rho\, dz = \frac{U\tau_{x0}}{M} \approx \frac{c_x \alpha U^3}{r\Pi_g}. \tag{11.17}$$

Formulas (11.2) and (11.15)-(11.17) now make it possible to obtain all the estimates desired. In particular,

$$U \approx \frac{(\varkappa - 1)^{-2/5} k^{1/5}}{\alpha^{1/5} c_x^{3/5}} \Pi_g^{2/5} \Pi_M^{1/5} \Pi_\omega^{1/5} c, \tag{11.18}$$

$$V \approx \frac{k^{2/5} \varkappa^{8/5} c_x^{1/5}}{(\varkappa - 1)^{8/5}} \frac{\Pi_M^{4/5}}{\Pi_g^{1/5} \Pi_\omega^{3/5}} c, \tag{11.19}$$

$$\delta T \approx \frac{(\varkappa - 1)^{9/10}}{k^{3/5} \alpha^{3/5} c_x^{1/5}} \Pi_g^{1/5} \Pi_M^{1/5} \Pi_\omega^{3/5} T_e. \tag{11.20}$$

The basic contribution to the total kinetic energy of circulation will be made by the zonal component. Hence $E \approx 2\pi r^2 M U^2$. The ratio of the energy to the value of E_0 yielded by formula (5.12) will be

$$\frac{E}{E_0} \approx \frac{(\varkappa - 1)^{1/5}}{a^{3/5} k^{1/10}} \left(\frac{\Pi_g}{c_x}\right)^{4/5} \frac{\Pi_\omega^{2/5}}{\Pi_M^{1/5}}. \tag{11.21}$$

The degree of ordering, β, is found to equal $\beta \approx \Pi_g/c_x\alpha$, that is, again proves to be of the order of unity.

If the values of the parameters corresponding to Earth are inserted into (11.21), we obtain $E/E_0 \approx 4$. At the same time, function $f_1(\Pi_\omega)$ estimated on the basis of (10.2) with $a \approx 1$ yields $E/E_0 \approx 3$ for Earth. In precisely the same way formula (11.20) overstates the actual temperature difference, δT, by a factor of approximately 2. The cause of these discrepancies is clear: we have failed to allow here for the meridional heat transfer in the atmosphere due to large-scale eddies, and as a result the values of U and δT, which are related to each other exclusively by means of the Ekman wind shift in the boundary layer, is inevitably overstated in comparison to the case in which the mechanism of transfer in the free atmosphere is also included. However, formulas (11.18)-(11.20) may be of interest as extreme estimates.

A.3. Boundary Rapid Rotation Conditions

As the speed of rotation of a planet increases, according to (11.18)-(11.20) U and δT will increase and V will decrease:

$$U \sim \omega^{1/5},\ V \sim \omega^{-3/5},\ \delta T \sim \omega^{2/5}.$$

At the same time, for δT there is the obvious restriction of (8.9), which /47
indicates that the atmospheric temperature in the coldest parts of the atmosphere cannot drop below the condensation temperature of the gases saturating the atmosphere, owing to the release of latent heat of condensation. For this reason the formulas in question cease to be accurate at large values ω. With increase in ω the meridional exchange is increasingly impeded, and this should lead to decrease in the temperature difference between the equator and the poles δT, but then the actual temperature dissipation will be nearer that determined by radiation alone, that is, equation (5.8) ceases to be valid. However, a maximum estimate of the mean speeds may be obtained even without this equation.

Since in the case of very rapid rotation it is to be expected that $\delta T/T_e \approx 1$ (see the end of Section 8), we write formula (11.2) in the form

$$\varepsilon \approx \eta_p \frac{q}{M}, \tag{11.22}$$

in which $\eta_p = k\delta T_{max}/T_e$. From (11.22) and (11.17) we immediately find

$$U \approx (\varkappa - 1)^{-1/5} \left(\frac{\eta_p \Pi_g \Pi_M}{c_x \alpha}\right)^{1/5} c, \tag{11.23}$$

from which, since $\eta_p < 1$, we obtain

$$U < (\varkappa - 1)^{-1/5} \left(\frac{\Pi_g \Pi_M}{c_x \alpha}\right)^{1/5} c. \tag{11.24}$$

For Earth, if it were to rotate much more rapidly with the values of the other parameters constant, that is, $\Pi_g \approx c_x \approx 10^{-3}$, $\Pi_M \approx 10^{-3}$, and $c = 300$ m/sec, in accordance with (11.24), $U < 40$ m/sec.

By means of (11.15) it is also possible to estimate the mean meridional velocity component V: on the increase in the speed of rotation it will decrease as ω^{-1}. No quantitative conclusions may be drawn regarding the behavior of characteristic temperature difference δT. It may only be anticipated that with accelerated rotation the poles will be colder and the temperature at them will approach a value determined by the local radiation equilibrium conditions.

B. Dissipation in the Free Atmosphere

B.1. Slow Rotation ($\Pi_\omega \ll 1$)

In keeping with Kolmogorov's idea (1942), the specific dissipation of kinetic energy may be estimated by means of the formula

$$\varepsilon \approx \frac{U'^3}{L}, \tag{11.25}$$

in which U' is the characteristic speed in large-scale disturbances of extent L. In the case of slow rotation U' will be commensurable with mean wind speed U, and L with the radius of the planet r. Then by means of (11.1), (11.2), and (11.25), we obtain formula (5.12):

$$E \approx 2\pi k' \beta c_p q r.$$

The derivation of this expression for the total kinetic energy is entirely equivalent to that used in Section 5. In this case, in accordance with (8.7), $\beta = 1$. /48

B.2. Rapid Rotation ($\Pi_\omega > 1$)

In this instance the spatial dimensions of the basic disturbances are estimated (obviously as a maximum) by the Obukhov scale, which it is convenient to apply here as

$$L = \frac{c}{\omega}. \tag{11.26}$$

The typical speed in such disturbances may be estimated on the basis of the mixing path theory:

$$U' \approx L\frac{\partial U}{\partial y} \approx L\frac{U}{r} = \frac{cU}{r\omega} = \frac{U}{\Pi_\omega}, \tag{11.27}$$

in which y is the coordinate in the meridional direction. Inserting (11.26) and (11.27) in (11.25), we obtain

$$\varepsilon \approx \frac{c^2 U^3}{\omega^2 r^3} = \frac{U^3}{\Pi_\omega^2 r}. \tag{11.28}$$

The regular meridional component of speed should be very small in the case of rapid rotation, since, if the wind shift in the Ekman boundary layer is disregarded, the basic heat transfer will be accomplished by large-scale eddies. This process may be described in simplified fashion by means of modified equation (5.8):

$$c_p M \frac{\partial \overline{U'T'}}{\partial y} \approx q \tag{11.29}$$

(T' is the temperature disturbance, the bar over it denoting averaging), in which the large-scale eddy heat transfer on the average for the entire thickness of the atmosphere approximately equals the flux of escaping radiation.

Estimating T' on the basis of the formula $T' \approx L\delta T/r$, which is analogous to (11.27), and assuming that the correlation between U' and T' is sufficiently large[5], we reduce equation (11.29) to the form

$$Mc_p UL^2 \frac{\delta T}{r^3} \approx q. \tag{11.30}$$

Formulas (11.2), (11.28), and (11.30) form a closed system for determination of the characteristics sought. In particular, from the system we find:

$$U \approx (ka)^{1/4} \Pi_M^{1/4} \Pi_\omega c, \tag{11.31}$$

$$E = 2\pi r^2 M U^2 \approx 2\pi (ka)^{1/4} \sigma^{1/4} c_p^{-1/2} q^{1/4} r^5 \omega^2. \tag{11.32}$$

Comparing the formula last given with (5.'2) or (8.1) and taking (8.3) into account, we obtain /49

$$\frac{E}{E_0} \approx \Pi_\omega^2. \tag{11.33}$$

This formula provides additional justification for relation (10.2) and shows that the latter may be valid both when $\Pi_\omega \ll 1$ and over a certain range of values $\Pi_\omega > 1$.

For δT and ε we obtain the expressions

$$\delta T \approx \frac{\Pi_M^{1/2} \Pi_\omega}{(ka)^{1/4}} T_e, \tag{11.34}$$

$$\varepsilon \approx (ka)^{3/4} \Pi_M^{1/2} \Pi_\omega \frac{q}{M}, \tag{11.35}$$

according to which both values increase with increase in Π_ω. The formulas in question may not be accurate at very large values Π_ω, since otherwise statement of inequality (8.9) would be violated (the situation is the same as in Section A.3).

From (8.7), (11.31), (11.32), and (11.35), we obtain the expression

$$\beta \approx \Pi_\omega^2, \tag{11.36}$$

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[5]A discussion of the role of correlation coefficient $k_{U'T'}$ is given by Golitsyn and Zilitenkevich (1972).

that is, β in this particular model depends only on Π_ω and increases with acceleration of rotation, as was predicted qualitatively in Section 8 (see the note on page 37).

B.3. Limiting Rapid Rotation Conditions

By means of formulas (11.22), (11.25)-(11.27) we find

$$U \approx \eta_n^{2/5} \Pi_M^{-1/5} \Pi_\omega^{2/5} c, \tag{11.37}$$

from which we derive

$$E \approx 2\pi \eta_n^{4/5} \sigma^{1/5} c_p^{-4/5} q^{4/5} M^{3/5} r^4 \omega^{4/5}. \tag{11.38}$$

Comparing this equation with (5.12) we obtain

$$\frac{E}{E_0} \approx \frac{\eta_n^{4/5}}{k^{1/2}} \frac{\Pi_\omega^{4/5}}{\Pi_M^{2/5}}. \tag{11.39}$$

Hence, in contrast with the results of the theory of similarity and the preceding subparagraphs of part B of this section, the total kinetic energy of the atmosphere is again found to depend on the mass of the atmosphere under limiting conditions, as was the case in subparagraph 8.3. The same considerations advanced in subparagraph 8.3 may be advanced here for the values of δT and ε.

CHAPTER 3. APPLICATION OF THE THEORY TO VARIOUS PLANETS OF THE SOLAR SYSTEM AND TO THE SOLAR ATMOSPHERE /50

12. Earth

The circulation of the atmosphere surrounding us is the only item about which we have quantitative information obtained with any degree of accuracy from observations (see the beginning of Section 3). However, as we shall see later, this accuracy leaves much to be desired. For the rapidly rotating Earth $\Pi_\omega = 1.43$ according to Table 3. Hence in formula (8.1) coefficient B incorporates function $f_1(\Pi_\omega)$. The available data permit estimation of B_0 or k and $f_1(\Pi_\omega = 1.43)$, and also of β separately. Consequently, although Earth is not the most suitable body for estimation of the constants of the theory, we have nothing better to work with.

The value of k may be found from expression (5.4):

$$k = \frac{T_1}{\delta T}\frac{\varepsilon M}{q} = \frac{r_1}{\eta_{id}}. \tag{12.1}$$

Let us turn first of all to the distribution of temperature in the atmosphere of Earth. The book by Lorenz (1967) presents meridional sections of the mean temperature plotted by Palmen and Newton (1969) on the basis of a large volume of experimental data. From these sections it is possible to compile a table of temperature values for the North Pole (NP), equator (E), and South Pole (SP) for two isobaric surfaces: 1,000 mb (surface of the Earth) and 5,000 mb (mean level of the atmosphere) and two characteristic months of the year (Table 5).

It follows from this table that $T_1 = 300$ K, on the average for the year $\delta T_s = 45$ K, $\delta T = 35$ K. Then $\eta_{id} = 45$ K/300 K = 0.15. We may note that we would commit only a small error if we were to assume that $\eta_{id} \approx \delta T/T_e = 35$ K/25 K = 0.14. The value of η_{id} may be assumed to be fairly definite, something which cannot be said of the rate of dissipation or generation of kinetic energy mass ε. A summary of the dissipation values in the unit atmosphere column and /51
on the average for the entire atmosphere is given by Lorenz (1967), where it is designated as D and is given in wt/m^2. It may readily be seen that this value is approximately equal (only for Earth, where $M \approx 10^3$ g/cm^2) to the value of ε in cm^2/sec^3, that is, the dissipation per unit mass. Determination of the total mean value of D from the empirical data is a difficult task owing both to the incompleteness of the data themselves and to the absence of direct methods of calculating D. Thus usually it is not D that is calculated, but rather the value of C, the rate of conversion of total potential energy to kinetic energy, which equals D on the average over a fairly lengthy period.

TABLE 5. CHARACTERISTIC TEMPERATURE (IN KELVIN) OF THE EARTH'S ATMOSPHERE

Isobaric surface, mb	NP	E	SP	NP	E	SP
	January			July		
1000	245	300	265	273	300	240
500	230	273	238	250	267	225

The estimates of Brunt (1926), Oort (1964), and Kung (1966) are noteworthy: ε equals respectively 5; 2.3; and 7.1 cm^2/sec^3. It is true that the last value was obtained exclusively on the basis of data above North America. Wiin-Nielsen (1968) subsequently obtained the value $\varepsilon = 6.4$ cm^2/sec^3. We may further mention (see Section 5) the estimates of Palmen (1959), according to which the mean rate of generation of kinetic energy of the atmosphere equals 4 cm^2/sec^3. In order to be definite we will select precisely this mean figure, but it must be remembered that the error may reach as much as ±50%. Then $\eta = \varepsilon M/q = 1.8 \cdot 10^{-2}$, that is, $k = 0.12 \approx 0.1$.

As has already been pointed out, the value of the total kinetic energy of the atmosphere apparently undergoes seasonal fluctuations. We follow Oort (1964) and set $E = 7.5 \cdot 10^{27}$erg, and in doing so we according to Lorenz run the risk of committing an error of about 20% or more. At this value of E the mean rate over the entire atmosphere equals 17 m/sec. Using this value of E and the value $E/B = 7.7 \cdot 10^{27}$ erg given in Table 4, we find that $B = B_0 f_1(\Pi_\omega) =$ $= 0.97$. Assuming B_0 and k to be connected by relation (8.2), we find that $B_0 \approx k^{1/2} = 0.32$, that is, $f_1(\Pi_\omega = 1.43) \approx 3$.

Let us further determine the value of ordering parameter β on the basis of formula (8.7). For the values adopted by us $\beta = 2.0$. The characteristic "lifetime" of circulation $\tau_0 = E/\epsilon = \beta r/2U = 3.4 \cdot 10^5$ sec = 4 days, that is, if the supply of energy to the atmosphere were to be discontinued, and if ratio $E/(\partial E/\partial t) = E/\epsilon$ were to remain unchanged, in four days the energy of circula- /52
tion would decrease by e times.

Establishment of the values of the constants of the theory and the approximate course of function $f_1(\Pi_\omega)$ makes possible a rough presentation of the possible modifications of the Earth's climate on change in one or another "external" parameter. Our approach is, of course, a highly simplified one which fails to allow for the numerous feedbacks existing in the atmosphere (see Monin, 1969), such as that regulating the role of evaporation and cloudiness, but all this probably is essential in the case of not overly abrupt changes in the "external parameters". In the case of extensive modifications our theory can probably provide a more or less correct answer.

We have seen that one of the basic factors determining the general circulation and climate is the spin velocity of the planet ω. We know that ω decreases constantly for Earth as the result of the tidal evolution of the Earth-Moon system (Monin, 1972). At the present time the length of the day is increasing by 1.7 m/s per hundred years. Around one billion years ago the day was approximately 5 hours shorter, and it is possible that several billion years ago there was in the history of Earth a brief period during which the days lasted about 5 hours. On the other hand, in $5.3 \cdot 10^9$ years, according to MacDonald (1964), the length of the day will reach one month, after which the month will become shorter than the day, and tidal friction will cause the Moon to fall to Earth in a rather short period ($7 \cdot 10^8$ years).

If the composition of the atmosphere and the other astronomical factors are assumed to be invariable, it is easy to follow the evolution of climate caused by rotation. Let us take formula (10.2) with the value a = 1. Then in the past the winds must have been much stronger and the poles appreciably colder[6]. On the other hand, warming of the polar regions is to be anticipated in the future, since $\delta T \sim f_1^{3/2}(\Pi_\omega)$. The maximum decrease in $\delta T = 3^{3/2} \approx t$, that is, the temperature difference between the equatorial and the polar regions will be only approximately 10 K. The minimum mean winds will be approximately 9 m/sec, while at the present U = 17 m/sec.

For Earth the criterion of occurrence of a particular type of circulation G as defined by formula (10.4) at the present time equals 0.07, that is, no substantial role is played by the temperature difference between the daytime and the nighttime hemispheres. However, when in several billion years the day comes to equal 2 weeks or more the situation will change; during the lengthy weeklong night the atmosphere will cool by more than 10 K and the circulation will be oriented from the daytime toward the nighttime side. Increase in the circulation space scale to Πr also results in appreciable intensification of the wind.

13. Mars /53

Let us assume that we are living in 1964, that is, that we possess the astronomical data on the planet and have just learned from the flight of Mariner-4 that $p_s \approx 5$ mb and that the atmosphere consists chiefly of CO_2. What may then be derived from the similarity considerations? For Mars $\Pi_M \approx 3.3 \cdot 10^{-2}$ and $\Pi_\omega = 10.5$ at this figure. At first we will set rotation aside, assuming k = 0.1. We may note that, according to the results of Section 9, $P_m = 0.033$ is not so small, and consequently the influence of decrease in function $f(\Pi_m)$ and the utilization factor may prove to be substantial. Hence the estimates

[6]However, it must be borne in mind that during the polar night the temperature at the high latitudes cannot drop below -183°C, the temperature of liquefaction of oxygen (or slightly lower than the liquefaction temperature of nitrogen). The situation will be the same as on Mars. The release of heat of condensation prevents further lowering of the temperature (see the end of Section 8).

made in what follows on the basis of the formulas in Section 8 may be regarded as exaggerated. Then with rotation not taken into account we obtain $E \approx 2 \cdot 10^{26}$ erg, $U \approx 45$ m/sec, $\tau_U \approx 1$ day, $\varepsilon \approx 150$ cm^2/sec^3, and $\delta T \approx 75$ K at the mean level of the atmosphere. Taking $\Pi_0 \approx 1$, that is $f_1(\Pi_\omega) \approx 2$, into account, we might say that the winds there are surely around 60 m/sec and that $\delta T \approx 100$ K, and are known to be more than 100 K on the surface. This suggests that the polar caps may consist of dry ice.

The calculations by Leovy and Mintz (1966) show that our predictions are not too far from their results. It is true that we have exaggerated the speed by a factor of approximately 1.5, but when it is remembered that estimates of the kinetic energy of circulation differing by a factor of 2 are encountered even for the Earth's atmosphere (see Lorenz, 1967), this forecast for Mars may be acknowledged to be satisfactory. In addition, our estimates are probably exaggerated.

Let us examine in greater detail the results of the second numerical experiment conducted by Leovy and Mintz (1969) on simulation of the circulation and climate of the Martian atmosphere. In this experiment calculations were performed for detailed maps of the distribution of wind, pressure, and temperature, and all the integral and mean characteristics we require are given: total kinetic energy of circulation, efficiency of the atmosphere η, and the mean meridional temperature sections. All the calculations were carried out for the winter solstice in the northern hemisphere and the autumnal equinox in the southern hemisphere. In consequence of the ellipticity of the orbit of Mars, the heat flux values differ appreciably in these two cases. The pressure at the surface was taken as equalling 7.5 mb, the partial pressure of CO_2 being 5 mb and nitrogen accounting for the remainder. The initial temperatures in the atmosphere and on the surface were assumed to equal 200 K.

The calculation of circulation was performed for 25 days.

Condensation of the atmospheric carbon dioxide (at 146.4 K, if $p_s = 7.5$ mb) began during the period of the equinox in both polar regions on the surface of the planet, and during the period of the solstice in the vicinity of the North Pole; a more or less stationary level of kinetic energy of circulation was established on the tenth day. However, strictly stationary circulation is not to be assumed, since in winter the mean atmospheric pressure decreases owing to condensation of CO_2 at a rate of 0.011 mb per day, that is, during the Martian /54
winter 20-25% of the entire atmosphere may settle in the polar cap. The latter situation is in agreement with the simple estimates made by Leighton and Murrey (1966), who substantiated the idea that the polar caps may consist of dry ice. The rate of condensation is an order of magnitude lower during the period of the equinox. Since at least one of the caps is always in existence on Mars, the actual amplitudes of fluctuations in atmospheric pressure is apparently less than 20%. The large flux of mass toward the cold pole and the large meridional temperature gradients in the winter hemisphere, owing to the Coriolis force, result in the development there of strong winds. In the summer hemisphere the

winds are slight owing to the small temperature gradients. The nature of temperature distribution and the varying condensation rate represent the chief causes of the difference in circulation characteristics during the two periods under consideration.

The basic mean circulation characteristics with which we are concerned are given in Table 6 for the periods of the solstice (S) and the equinox (E).

TABLE 6. BASIC THEORETICAL CHARACTERISTICS OF CIRCULATION OF MARTIAN ATMOSPHERE FOR TWO PERIODS (ACCORDING TO LEOVY AND MINTZ, 1969).

Period	$q \cdot 10^{-3}$ g/sec^3	[illegible] erg	T, K	ΔT, K	[illegible]	cm^2/sec^3
S	7,24	1,46 1,66	250	100	2,6 10[illegible]	20
E	5,61	0,47 0,53	215	65	1,2 10[illegible]	7

Commas indicate decimal points.

Attention is attracted by the fact that the value of the utilization factor is more than an order of magnitude smaller than the corresponding factor for the Earth's atmosphere. This is due to the poor "greenhouse" properties of the Martian atmosphere, as a result of which it emits a greater part of the solar energy whereever the latter is received, and the thin atmosphere of the planet is not capable of accomplishing spatial transfer of a large quantity of heat. This reduces the effectiveness of the atmospheric heat tension.

Table 7 presents values calculated on the basis of the data in Table 6 for the degree of flux ordering β, utilization factor κ, and the value of function $f_1(\Pi_\omega)$ for the same periods found on the assumption that $P_0 \approx k^{1/2}$ and $B = B_0 f_1(\Pi_\omega)$. /55

The data in Table 7 show that though the results of the calculations by Leovy and Mintz (1969), owing to the specific features noted of the Martian atmosphere[7], are not ideal for estimating the coefficients and values of function $f_1(\Pi_\omega)$ introduced here, yet the dispersion of the latter does not exceed the accuracy of the known characteristics of Earth's atmosphere. Thus we take $\beta \approx 2.5$, $k \approx 5 \cdot 10^{-3}$, $f_1(\Pi_\omega) = 1.9$ as the mean values, the seasonal variations probably being realistic.

[7]Let us consider yet another circumstance in this context. Use of formula (8.8) to estimate δT with the values of f_1, β, and k found for the solstice would yield $\delta T \approx 200$ K, that is, $T_2 \approx 50$ K. However, the condensation beginning at $T_c \approx 150$ K prevents the atmosphere from cooling below this temperature because of the release of latent heat. In the case of the equinox, when condensation plays a minor role, all the values determined are found to be mutually consistent and agree fairly well with the theoretical ones.

Thus we know the values of function $f_1(\Pi_\omega)$ at three points: at $\Pi_\omega << 1$ $f_1(\Pi_\omega) = 1$, for Earth $f_1(\Pi_\omega = 1.43) \approx 3$, and for Mars $f_1(\Pi_\omega \approx 1.05) \approx 1.9$. This behavior may be approximated by a relation of the form $\varphi_1(\Pi_\omega) \approx 1 + a\Pi_\omega^2$, in which $a \approx 0.9 \pm 0.2 \approx 1$, this being in agreement with (10.2).

To conclude our discussion of the results obtained by Leovy and Mintz (1969) we may note that their experiments demonstrate the presence of a large diurnal component in the wind field caused by the sharply defined boundary between the dark and illuminated sides of the planet. The authors term it the diurnal thermal tide. The existence of this effect on Mars can also be easily understood within the context of our concepts. According to the results of Section 10, the relative role of such effects for the general circulation is estimated as the value of dimensionless criteria G defined by formula (10.4). For Mars $G \approx 0.5$, that is, it is to be anticipated that the diurnal variation of temperature on the planet must as a matter of fact play an appreciable role in determining the general circulation conditions of the Martian atmosphere.

In all these calculations and estimates no allowance is made for the surface topography of Mars, which the data obtained from space stations and by radar show to be highly complex. Topography heavily influences the nature of winds; for this reason the speeds of the actual local winds may differ appreciably from the estimates indicated here. Qualitative estimates of the role of topography in the wind conditions on Mars have been made by Gierasch and Sagan (1971) and by Sagan, Veverka, and Gierasch (1971). In a book published in 1973 at least brief mention should be made of the results obtained in 1971-1972 by the unmanned interplanetary stations Mariner-9, Mars-2, and Mars-3. Although the bulk of the scientific data still had not been processed by May of 1972, let alone published, it is obvious that these results will lead to a gigantic increase in our knowledge about Mars. The observations on the topo- /56
graphy and configuration of the planet, the composition of its atmosphere, the atmospheric temperature profiles, the structure of the upper atmosphere, the magnetic field, and so forth will all greatly broaden our ideas about the red planet.

TABLE 7. THEORETICAL CIRCULATION CHARACTERISTICS

Period	β	k	$f_1(\Pi_\omega)$
S	2,4	$6{,}5 \cdot 10^{-3}$	2,4
E	2,6	$4 \cdot 10^{-3}$	1,3

Commas indicate decimal points.

The beginning of exploration of Mars by means of space stations coincided with a dust storm of enormous intensity which lasted for around 4 months. In this book Section 19 is devoted to the question of the dust storms, since consideration of the latter requires allowance for many factors, including local ones, which we study in the next chapter. Here we shall confine ourselves merely to presenting a few of the most interesting television photographs of

Mars obtained by Mariner-9. Dust clouds (Figure 3) and other meteorological phenomena are to be seen in them, in particular the undulating cloud systems beyond the mountains (Figures 4, 5) and the dynamics of melting of the southern polar cap (Figure 6). Interesting photographs depicting the manifestations of dynamic phenomena in the Martian atmosphere are also presented in the article by Leovy et al. (1972). /57

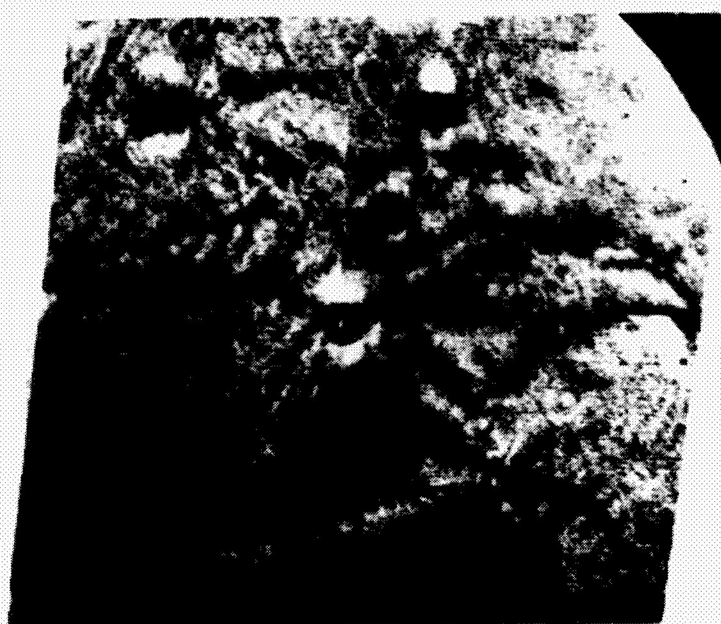

Figure 3. Last Television Photograph of Mars Immediately Before Transfer of the Mariner-9 Station to Orbit Around the Planet on 13 November 1971. The surface is concealed by a dust storm and all the details are those of the cloud cover. The four spots are probably high craters. The southernmost crater is around 200 km in diameter and is situated somewhat below the equator. The edge of the planet is at the upper right. The photographs of Figures 3-6 were kindly made available by S. I. Rasool.

14. Venus /59

The data in Table 4 show that, according to our estimates, $E \approx 5 \cdot 10^{27}$ erg. Then with the mass of the atmosphere of the planet Venus $M_0 \approx 5 \cdot 10^{23}$ g (two orders of magnitude larger than Earth), U should be around 1.5 m/sec. At the same time, the criterion of the form of circulation (9.3), the G number, for Venus is found to be near 2. This indicates that the main cause of circulation may be represented by the temperature difference between the dark and the light sides of the planet, and for this reason our speed estimate may be underestimated. Calculations performed by Zilitenkevich et al. (1971), and by Turikov and Chalikov (1971) on the basis of a two-level model confirmed this assumption. The circulation is found to be rather asymmetrical owing to the large thermal inertia of the atmosphere of Venus, the center of action with high pressure being shifted 45° in longitude toward the evening terminator and the center of action with low pressure being situated in the vicinity of the morning terminator. Calculations for an atmospheric model involving absorption of the greater part of the apparent radiation of the surface of the planet have yielded mean wind speeds of the order of 5 m/sec, while the typical temperature difference at the surface has been found to be of the order of 1 K. Somewhat lower speeds and slightly larger temperature differences have also been obtained in a circulation model in which all the apparent radiation is absorbed in the upper theoretical layer of the atmosphere. Numerical calculations and comparison of the results of the latter with our results demonstrate that utilization factor k for Venus must be near unity (see Turikov, Chalikov, 1971). /60

Figure 4. Complex of Craters in the Region of Tarsis (Southern Hemisphere of Mars). The photograph was taken on 28 November 1971. The main crater has a diameter of around 20 km. When Mariner-9 approached Mars this region was visible as a dark spot penetrating the dust clouds. The light arcs reflect the structure of the cloud cover.

Thus the basic conclusion regarding Venus is a high degree of homogeneity of temperature distribution in its atmosphere, this being due to the great mass of the atmosphere. The mean speeds in the main layer of the atmosphere of the planet are of the order of several meters per second.

The history of study of the atmosphere of Venus is one of the most instructive ones in the development of science. The fact of existence of an atmosphere on this planet was established so long ago as 1761 by the great Russian scientist M. V. Lomonosov, who observed the passage of the planet across the disc of the Sun. At the moment of contact between the planet and the disc of the sun he noted that the latter was elongated in the direction of the planet and enveloped it. This was interpreted by Lomonosov as the effect of refraction in the atmosphere of the planet. This effect now bears his name.

Figure 5. Crater in the Northern, Winter, Hemisphere of Mars Taken Approximately Four Months after the Dust Storm Ended. It is around 40 km in diameter. The edges of the crater are surrounded by hoar frost. Beyond the edges of the crater the wind generates waves of clouds, which can be traced downwind for hundreds of kilometers. The results of measurement of the vertical temperature profiles confirm the assumption that these clouds consist of particles of condensed carbon dioxide. The photograph of the same region taken approximately 17 days later (see Leovy et al., 1972) no longer shows these waves. This demonstrates that what we actually see here are atmospheric orographic waves.

In 1932 Danham and Adams (see Moroz, 1967) discovered in the vicinity of wavelength 0.8 μ in the spectrum of Venus two unknown absorption bands which were later identified with the absorption bands of CO_2.

An important discovery was made in the mid-1950s. It was found that the brightness temperatures of the planet in the radio range of the centimeter and decimeter waves is around 600 K. At that time it was difficult to reconcile oneself to the idea that such temperatures might apply to the surface of the planet or to its lower atmosphere. Thus until 1963 the most popular hypothesis was that the ionosphere of the planet is responsible for this radiation. However, observations conducted with Mariner-2 in 1962 and a number of others refuted this hypothesis. If the radiation had proceeded from the ionosphere then the scanning radiotelescope of Mariner-2 should have registered increase in the brightness of the disc of the planet toward the edge, while darkening was actually observed. In addition, according to these observations the temperatures of the dark and illuminated sides of the planet are virtually the same. It follows that Venus must have a deep atmosphere which is hot at the bottom. Definitive confirmation of this circumstance was obtained after the flights by the Venus-4 and Mariner-5 stations.

Measurement of the distributions of temperature over the disc of the planet also has a history of its own, one eloquently described, together with other studies of Venus, by Sagan (1971) in a survey entitled "The Troubles with

Venus". Up to the middle 1960s almost all researchers observed a large phase variation in the temperature of radiation on almost all radio wavelengths (also see Moroz, 1967) with amplitudes of the order of 50-100 K. This means that the surface or atmosphere temperature varies in this manner in the course of a day. However, measurements made at the end of the 1960s (see Sagan, 1971a) revealed the absence of a phase variation in radiation temperature on all the wavelengths used. /61

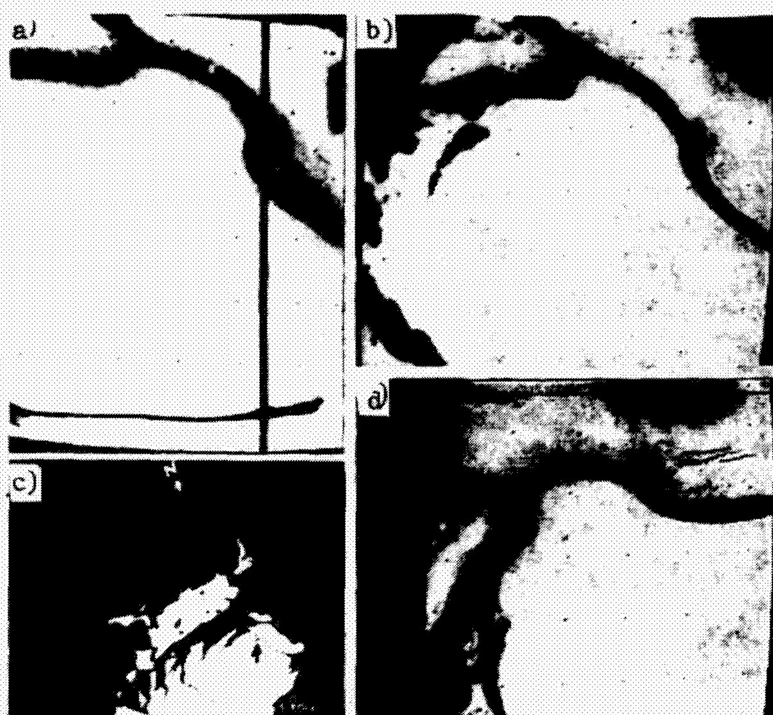

Figure 6. Three High-Resolution Photographs of the Region of the Southern Polar Cap of Mars, Which is Indicated by an Arrow in Photograph C. The region is about 100 km in diameter. Reduction in the area covered by the cap is to be seen. A layer of dry ice (carbon dioxide) only around 2.5 cm thick may evaporate in 12 days. a, 19 November; b, 28 November; g, 1 December 1971.

The distribution of the radio brightness temperature along the meridian was first obtained by Kuz'min and Clark (1965) by means of a radiointerferometer having a wavelength of 10.6 cm. They found that the poles are 150-200 K colder than the equatorial regions. However, measurements by Berge and Greisen (1969) on a wavelength of 3.12 cm demonstrated total absence of any difference between the brightness temperatures of the equator and the poles. On a wavelength of around 3 cm the atmospheric absorption is appreciable, and so the radio emission on 3.12 cm comes from the atmosphere itself, although from rather deep layers of the latter. If the measurements on the two wavelengths (10.6 and 3.12 cm) are accurate, the temperature distribution in the atmosphere of Venus must be characterized by enormous inversion in the polar regions. This fact represented one of the reasons moving the author to attempt to arrive at a theoretical understanding of the thermal conditions of the atmosphere of Venus. The effort ultimately led, in 1969, to construction of a theory of similarity for the general circulation of planetary atmospheres.

The findings of this theory were in agreement with the results of measurements on 3.12 cm. Insofar as Venus is concerned, this at the time was the only confirmation that the theory was correct. At the International Symposium on Planetary Atmospheres held in Martha, Texas, at the end of October 1969, at which the author for the first time presented a report on his theoretical results, it was ascertained that interference measurements on 11.1 cm (see Sinclair et al., 1970), which definitely pertained to the surface of the planet, the uniformity of the temperature distributions of Venus along the meridian is demonstrated, also within the limits of an error of around 12°. On the same occasion facts were presented which proved the absence of a phase variation in temperature on Venus, this also being in agreement with the concepts of our theory.

The flights by the Soviet unmanned space stations of the Venus series yielded the first direct information on winds in the atmosphere of this planet (Kerzhanovich, 1972; Kerzhanovich, Morov, Rozhdestvenskiy, 1972; Kerzhanovich et al., 1972). The method of wind speed measurement (Kerzhanovich et al., 1969) consists of determining the Doppler shift of the frequency of the radio transmitter of the station parachuted into the atmosphere of the planet. The basic shift results from the relative motion of Earth and Venus and their diurnal rotation. Fortunately, this motion is known with a very high degree of accuracy and may be excluded. If the station descends to an underground point situated on a line connecting the centers of Earth and Venus, the remaining Doppler shift is due to the vertical descent of the station. However, if the station descends to a point remote from the underground one, then it descends at a certain angle to the line of sight, and if it undergoes horizontal dis- /62
placement as a result of the action of wind, the speed of the wind will contribute to displacement of the frequency of the signal received[8]. The speed of purely vertical descent may be calculated with good accuracy if the vertical pressure and temperature profiles are known.

According to the data of the Venus-4 station (Kerzhanovich, 1972) at the point of its landing (night side of the planet, about 500 km north of the equator, near the morning terminator) over the 40-50 km altitude range winds blow in the direction of the equator at speeds of 40-50 m/sec. At lower altitudes the speeds decrease fairly rapidly and fall within the limits of the measurement errors, which are ± 10 m/sec. The data of the Venus-7 station (Kerzhanovich et al., 1972; Kerzhanovich, Morov, Rozhdestvenskiy, 1972), which were obtained by means of modified equipment, indicate that the zonal wind component ranged from 5 to 14 m/sec over the altitude range from 53 to 38 km.

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[8]The transverse Doppler shift in the ratio v/c is smaller than the longitudinal, in which v is the speed of the signal source and c is the velocity of light. Hence this effect is insignificant in movement along the line of sight.

Below 38 km the wind speed did not exceed 4 m/sec, a value comparable to the measurement and processing errors. In this sense the direct wind speed measurements at the point of landing of the Venus-7 station (the station landed near the equator and the morning terminator), as in the case of the Venus-4 station, do not contradict the estimates obtained on the basis of our theory[9].

In speaking of the winds on Venus, we must not fail to mention the so-called four-day circulation in the atmosphere of this planet. It was first discovered by Boyer and Camichel (1961) by photographing Venus in ultraviolet light (also see Smith, 1967). Against the general light background of the disc of the planet, there was observed a dark formation often having the form of the letter Y or Ψ lying on its side and with the central bar situated on the equator. This formation rotates about the planet with a period of around 4 days, this corresponding to wind speeds of around 100 m/sec. The circulation in this instance is purely zonal. The lifetime of the formations in question is on the order of a month.

Figure 7 shows photographs of Venus taken with different light filters and at consecutive time intervals. These photographs were made in the observatory of the New Mexico State University, USA, and were kindly made available to the author by Doctor B. A. Smith. The dark formations and their displacements over time over the disc of the planet are clearly to be seen in the ultraviolet rays. No details whatever are to be seen in visible light. The radius of Venus in ultraviolet rays, according to Kuiper (1971), is 6145 km, while according to his measurements in red light and in the near infrared region r = = 6,100 km. Kuiper assumes this distance to be real, and the ultraviolet dark clouds, as these formations are still called, are then found to be approximately 40-50 km above the visible yellow clouds. The atmospheric pressure at altitudes of 90-100 km from the surface of the planet is of the order of one mb (Avduyevskiy et al., 1969; Marov, 1971), that is, the four-day circulation is a stratospheric or even mesospheric phenomenon on Venus. /64

Attempts at hydrodynamic explanation of the four-day circulation have been undertaken by Schubert and Young (1970), Malkus (1970), and Gierasch (1970). The physical fact underlying these explanations was represented by an effect observed by Schubert and Whitehead (1969). These authors observed movements of mercury in an annular vessel occurring on heating of the liquid by a Bunsen

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[9]According to the data of the Soviet Venus-8 automatic space station (see Pravda of 10 September 1972), which landed on the light side of the planet and far from its underground point, along the path of descent of the station, the wind reached 50 m/sec in the atmosphere over the 40-60 km altitude range and dropped to 2 m/sec and below within the lower 10-12 km above the surface of the planet. The temperature at the point of landing equalled +470 ± 8°C, that is, differed hardly at all from the temperature of the planet's surface, +474 ± 20°C recorded by the Venus-7 station for nighttime (Marov, 1971). One of the most important scientific results obtained by the Venus-8 station is that it was possible for the first time to measure the vertical illumination profile. On the surface of the planet the illumination was found to be small, but was nevertheless a finite value.

burner flame moved in a circle below the bottom of the vessel. The mercury began to twist in the direction opposite of that of movement of the burner, the speed of movement of the liquid exceeding the speed of the burner in absolute value. In the case of Venus the role of the moving source of heat is performed by solar radiation absorbed by the atmosphere. This source completes a full revolution around the planet in 117 days — such is the duration of solar days on Venus — and the upper layers of the atmosphere accelerate in the opposite direction.

The following may be the hydrodynamic explanation of this effect. The heating of a liquid by a moving source of heat, owing to the finite value of thermal conductivity, lags in phase behind the movement of the source, this lag being the greater the farther are the layers of liquid from the source. Vertically inclined convection cells develop as a result in the case of a periodic source of heat. The Reynolds stresses occurring in this instance are such that the vertical flow of horizontal momentum $\tau = -\rho\overline{u'w'}$ (u' and w' are the zonal and the vertical velocity components) is directed upward. This causes increase in the mean wind profile with altitude until the viscous (turbulent) stresses balance the Reynolds stresses.

Schubert and Young (1970) demonstrated that the intensity of this effect depends on the value of dimensionless criterion

$$F = \frac{gH}{U^2}\frac{\Delta T}{T}, \tag{14.1}$$

in which g is the acceleration of gravity; H is the characteristic vertical scale of motion (in this instance the altitude of the homogeneous atmosphere); U is the speed of movement of the vertical source, that is, the speed of movement of the trail of the Sun over the surface of the planet determined by its spin and the rotation of the planet about the Sun; ΔT is the value of overheating of the liquid or the amplitude of the diurnal temperature variation referred to the characteristic temperature of the medium, that is, to the effective atmospheric temperature T_e. Since $gH \approx c_e^2$, and $U \approx \omega r$, this criterion may also be rewritten as /65

$$F = \frac{1}{\Pi_\omega^2}\frac{\Delta T}{T},$$

or, (10.3) and (10.4) being taken into account, as

$$F \approx \frac{\pi\Pi_M}{\Pi_\omega^3} = \frac{G\Pi_M^{1/2}}{\Pi_\omega^2}. \tag{14.2}$$

For Venus the value of this criterion is on the order of 100, for Earth 10^{-3}, and for Mars 0.1. From this Schubert and Young (1970) concluded that the effect under discussion may be manifested only on Venus. It could also occur on Mercury if this planet had any appreciable atmosphere. This effect is insignificant in the atmospheres of the other planets.

On the whole the theory of four-day circulation is still in the initial stage, in which the lack of observational data and the obvious complexity of the phenomenon permit tl e advancement only of simple qualitative ideas.

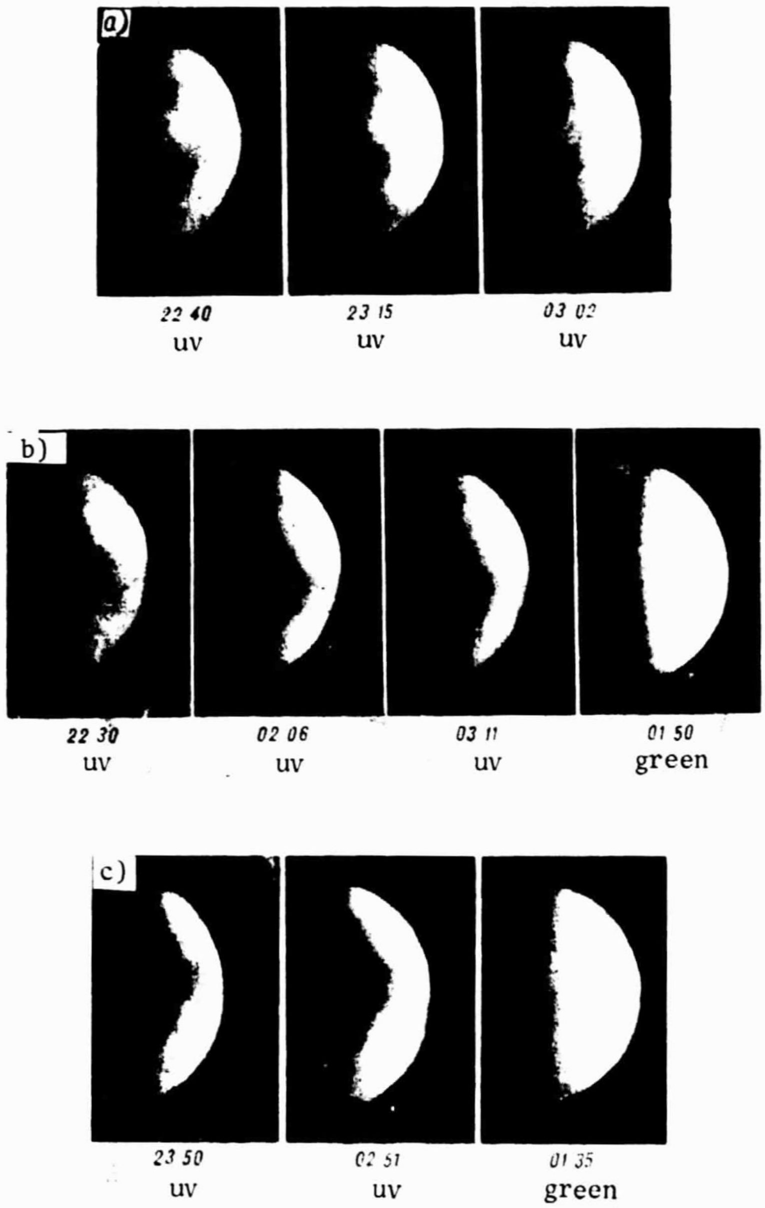

Figure 7. Photographs of Venus in Ultraviolet Rays and in Green Light. The dark formations shifting with time are to be seen in the photographs for which an ultraviolet filter was used. The figures below the individual photographs of the disc of the planet represent local time (hours and minutes). a, 21-22 May; b, 7-8 June; c, 13-14 June 1967. Photographs kindly sent by B. A. Smith. /63

15. Jupiter and Saturn

From the purely external standpoint, if the rings and the rare appearance of spots are disregarded, Saturn is very similar to Jupiter: the same dark belts and light zones parallel to the latitudes; the shorter period of rotation of the equatorial regions in comparison to the period of the temperate latitude. The values of the basic rotational similarity criterion Π_ω are also similar (see Table 3).

Winds have been observed for more than 100 years now on Jupiter at cloud level, on the basis of movement of the spots. A summary of the observations up to 1957 is given in the book by Peek (1958) (also see Moroz, 1967). A critical continuation to 1966 was made by Chapman (1969). If the distribution of the periods of rotation by latitudes are known, it is possible to estimate the mean kinetic energy of the zonal motions, on the assumption that the atmosphere is affected evenly by them to a depth characterized by quantity M. This energy may be estimated by use of the formula

$$E = \frac{\pi^4 r^4}{15 t_0^4} M \sum_i \cos^3 \vartheta_i \Delta\vartheta_i \Delta t_i^2, \tag{15.1}$$

which is obtained if it is considered that the speed of the ith zone relative to the mean rotation of the planet of period $t_0 = v_i = \Delta\omega_i r\cos\vartheta_i = 2\pi\cos\vartheta_i \Delta t_i t_0^{-2}$. In this equation ϑ_i is the mean width (in degrees) of the band or zone with /66 central latitude ϑ_i, and Δt_i is the difference between the observed period and t_0. Carrying out summation over all the bands and zones we find $E = 8 \cdot 10^{27}$ M erg $\approx 10^{28}$M erg, in which M is in g/cm^2. Almost 90% of this value is represented by this equatorial band and northern temperate current C.

According to Moroz (1967), the pressure at the cloud level is of the order of 1.5-2.5 atm. On the assumption that the motions affect a layer of the atmosphere of a thickness equal at least to the altitude of one homogeneous atmosphere, we obtain a minimum estimate of the value of $M = p/g > 10^3$ g/cm^2. Thus the total kinetic energy of circulation is $E > 2 \cdot 10^{31}$ erg.

We may note that, if no allowance were made for rotation, the estimate of E on the basis of (8.1) would yield only $E \approx 4 \cdot 10^{28}$ erg, that is, the role of rotation is actually the decisive one.

Even the minimum estimate obtained for energy E permits the making of a number of other useful estimates from the most general positions, since the total flux of energy toward the atmosphere is known to be $Q = 4\pi r^2 \sigma T \approx 1 \cdot 10^{25}$ erg/sec.

First of all we estimate the value of $\beta\eta$, which may be written by means of (7.7) and (3.4) as:

$$\beta\eta = \frac{2EU}{Qr}. \tag{15.2}$$

that is, we express the product of the degree of ordering of the flux by the atmospheric efficiency in terms of the more customary and observed quantities. The mean velocity of the visible motions across the disc are of the order of 30 m/sec. Then $\beta\eta > 2$. Since usually $\eta \ll 1$ and for large values M the value $\eta \approx M^{-1/2}$ (see Section 10), we should have $\beta > 2$, or $\tau_U \gg \beta r/2U = 7 \cdot 10^9$ cm; $3 \cdot 10^3$ cm/sec $\approx$ 1 month. Large lifetime values on Jupiter are typical even of minor details. According to Peek (1958), small spots are tracked on the disc of the planet for many weeks, months, and even years. This indicates a very low intensity of turbulence in the atmosphere of the planet[10].

The typical periods of variation in the width of the bands and zones on Jupiter, which may be regarded as periods of large-scale weather phenomena, may have a greater bearing on the question of the characteristic "lifetime" of circulation. A table of such periods prepared by Fokas is given in the book by Maroz. It follows from this table that the duration of these periods is chiefly 12-16 years, or $(4\text{-}5) \cdot 10^8$ sec, including the equatorial zone, in which the highest speeds are observed. However, even this time may prove to be substantially shorter than the circulation lifetime of $\tau_U = E/\eta Q$. As a matter of fact, if by the analogy with Earth we assume an efficiency of $\eta = 10^{-2}$, and this in all probability is a great exaggeration for the deep atmosphere of Jupiter, since $\eta \sim M^{-1/2}$, we will have $E = \eta Q \tau_U = 10^{25}$ erg/sec$\cdot 4 \cdot 10^8$ sec$\cdot 10^{-2}$ = /67
$= 4 \cdot 10^{31}$ erg, that is, a value comparable to our minimum estimate for the total kinetic energy of atmospheric circulation of the planet.

Much larger values of the energy of the motions, and consequently also of the depth of the atmosphere affected by them, and of the circulation lifetime, may be obtained by use of function $f_1(\Pi_\omega)$ in the form of (10.5). This has been done by the author (Golitsyn, 1970b). However, since relation (10.5) is hypothetical in nature, we will not consider this matter in detail.

Let us turn to Saturn. The period of rotation of the equatorial zone, which is ±20° in width, is near 10 hours 15 minutes, and the periods for the latitudes above ±30° is near 10 hours and 40 minutes. The kinetic energy of apparent motions relative to the middle latitudes may then be estimated on the basis of formula (15.1). It is found to be approximately 10^{29} M erg, that is, the value of the numerical coefficient of M is one order of magnitude larger than was the case for Jupiter. Such a difference is surprising. As a matter of fact, writing the expression for the total kinetic energy in the general form

$$E \quad 2\pi B_0 f(\Pi_\omega, \ldots)\, \sigma c_p\, T_e\, r \qquad (15.3)$$

[10]The rapid changes in shape or color of individual regions of the planet that are sometimes observed are probably due to condensation instability of the cloud systems caused by small variations in temperature (Moroz, 1967), something which has no direct bearing on the hydrodynamic stability of the flux.

(T_e is introduced into (8.1)) and assuming the ratio in the form of function $f(\Pi_\omega,...)$ to be on the order of unity for both planets because of the similarity of their Π_ω values, we find that the total kinetic energy for Jupiter should be approximately 5 times as great for Saturn. By comparing the estimates of E based on the apparent motions, we find from this that the atmosphere of Saturn is affected by motions to a depth based on mass that is approximately 50 times smaller than on Jupiter, although there are data (Moroz, 1967) indicating that the pressure at the cloud layer level of Saturn is also on the order of 1 atmosphere.

In view of this situation the possibility is not to be disregarded that the atmospheric dynamics of Saturn differ substantially in some respects from the dynamics of Jupiter. Thus in the functions $f(\Pi_\omega,...)$, although arguments Π_ω are similar, the other factors indicated by the dots may be entirely different, and then the ratio of these functions for the two planets differ greatly from unity.

There is reliable knowledge of only one substantial difference between Jupiter and Saturn. The former has a strong magnetic field of the order of 10 gauss, while in the case of Saturn all attempts to detect this field have not as yet been successful. It apparently is very weak or does not exist at all. Attention has repeatedly been called by Hide (see his survey, 1969, which also includes a large bibliography) to the possible importance of hydromagnetic effects, at least for sufficiently great depths of the atmosphere of Jupiter. Hence if may turn out that precisely Saturn is a "normal" rapidly rotating planet from the viewpoint of atmospheric dynamics, while intensive movements in the atmosphere of Jupiter are impeded by hydromagnetic interactions. If such is the case, if Jupiter had no magnetic field, the apparent motions in its atmosphere could be characterized by higher velocities, something which would reduce the difference in the values of the atmospheric mass affected by the /68
motions.

Jupiter and Saturn are also of great interest as purely hydrodynamic subjects. We may cite three important hydrodynamic problems for these planets which remain to be solved. They are the equatorial acceleration, the striped structure of the discs of the planets, and the nature of the Great Red Spot of Jupiter. All these problems have been discussed in detail by Hide (1966, 1969).

In order to explain the equatorial acceleration, it is necessary to find a mechanism of transfer of angular momentum from the temperate latitudes to the low ones. As has been demonstrated by Hide (1970b) and Starr (1971), such a mechanism can be provided only by asymmetrical disturbances of the zonal flux, but it is not as yet clear what the nature of such disturbances might be. Closely related to the foregoing is the problem of explaining the striped structure of the discs of Jupiter and Saturn. This structure unquestionably is a reflection of the structure of atmospheric circulation and some sort of instability of the latter. Attention was first called to this by Hess and

Panofsky (1951). Stone (1967, 1970) attempted to find new types of baroclinic instability in the zonal flux, including nongeostrophic ones, but the applicability of these results to the actual conditions is naturally always open to doubt.

A highly interesting work is that by Ingersoll and Kuzzi (1969), who calculated the relative speeds of the thermal wind, using the observed values of the albedo gradients, and finding very good agreement with the observed distribution of velocities along the meridian. It was found that the light zones must be warmer than the dark bands. These results are not in agreement with the assumption regarding monotonic variation in insolation with latitude as the sole source of the energy of the flows.

Of interest in this context is the hypothesis of Barcilon and Gierasch (to the effect that the banded structure of Jupiter reflects variation in the concentration of condensing substances with latitude). The moist adiabatic gradient varies owing to uneven release of the heat of condensation along the meridian, this circumstance causing a thermal wind observed as variations in the periods of rotation with latitude. The results obtained by Barcilon and Gierasch (1970) are in agreement with the results of Ingersoll and Kuzzi (1969).

There is an extensive bibliography on the question of the nature of the Great Red Spot on Jupiter. This spot was discovered by Hooke (1665). In the photograph of Jupiter (see Figure 1) it is situated in the southern hemisphere.

The Great Red Spot was barely noticeable during the 18th and for a large part of the 19th century, apparently owing to the low contrast, but about 100 years ago it abruptly became darker. The Great Red Spot is usually elliptical in shape and extends 20-40,000 km in longitude and 5-10,000 km in latitude. The period of rotation of the Great Red Spot has changed irregularly over the last 100 years. It has differed from the period of rotation of the temperate latitudes, so that the total difference in 100 years has been of the order of /69
6π, that is, over this period the Great Red Spot has gone around the disc of the planet approximately 3 times. Hypotheses to the effect that it is volcanic in origin or that it is an island floating in denser atmosphere, which were advanced up to the middle of the 20th century, are for various reasons unsatisfactory (see Moroz, 1967). Hide (1961) advanced a new hypothesis according to which the Great Red Spot is a so-called Taylor column, a disturbance in the geostrophic flow created by unevenness on the hard surface of the planet. This hypothesis was the subject of lively discussion during the 1960s (a survey of the literature is to be found in Hide, 1969), but it encounters the same difficulty: the speed of rotation of the solid body of the planet must be assumed to be uneven.

Our estimates of the circulation characteristics for Jupiter indicate that the circulation lifetime must be very large. Estimates of τ_U with function $f_1(\Pi_\omega)$ on the basis of (10.5) revealed (Golitsyn, 1970b) that τ_U may be of the order of millions of years. It may be assumed that the Great Red Spot is simply a large atmospheric vortex. Thus it need not be a permanent feature of the planet. An hypothesis such as this is free of a number of the defects of previous hypotheses (see the discussion of it by Sagan, 1971 b); however, it is too general and does not explain the variations of the different periods in the form and position of the Great Red Spot.

The theory of the Great Red Spot which is the most complete up to the present, one which explains the majority of the details of its variation in shape, displacements in longitude and latitude, and so forth, has been proposed by Street, Ringermacher, and Veronis (1971). The authors have advanced the hypothesis that there is a large accumulation of condensed matter, such as solid hydrogen, floating in the interior of the planet. There would then be a Taylor column above it. Any vertical displacements of this accumulation would result in melting or growth of the Great Red Spot, as a result of which its shape would also change. As a result of preservation of the momentum of the accumulation, horizontal displacements would lead to fluctuations relative to its mean position. The density of this accumulation is practically equal to the density of the ambient medium, and it behaves as does a float inside a stratified liquid. Such a float was first invented by Descartes, so that this theory of the Great Red Spot is known as the hypothesis of the Descartian slope. Despite the attractiveness of this theory, the nature of such a possible accumulation of matter deep within the planet remains vague, since the results of Trubitsyn (1972) definitely indicate the impossibility of condensation of hydrogen to the solid phase.

Cyclonic circulation of gaseous masses exists in the region of the Great Red Spot; it has been very effectively traced by Reese and Smith on the basis of movement of a small spot over the disc of Jupiter. The positions of this small spot relative to the Great Red Spot itself are indicated by boldface dots in Figure 8. The figures with the dots are the dates in January of 1966 when this picture was observed. By using these data, Hess (1969) estimated that the /70
mean vorticity within the Great Red Spot, as determined on the basis of the circulation value, equals 10^{-5} sec^{-1}, and the corresponding Rossby number is accordingly near 0.08. The values of the two quantities are of the order of such quantities for large-scale motions in the Earth's atmosphere.

16. The Sun

The Sun, the source of energy for all atmospheric motions, itself possesses a broad spectrum of motions on the apparent surface of its disc. We shall here consider only the motions of the largest scale, ones which the meteorologists would term general circulation. Among the great meteorologists who have concerned themselves with the nature of motions on the Sun mention may be made of Wilhelm Bjorknes. An entire chapter in the book by Starr (1968) devoted especially to this problem has been written in the language of contemporary meteorology.

The Sun, which is the star closest to us, has an apparent disc of a radius equalling 696,000 km. This is the upper boundary of the so-called photosphere, the irradiating surface of the star. The acceleration of gravity at this level equals 27.4 m/sec^2. The greater part of the solar matter is represented by ionized hydrogen — a mixture of protons and electrons with a certain admixture of helium, and to a much lesser extent, other heavier elements. Thus it is usually assumed that the molecular weight is $\mu = 0.6$ (see de Jager, 1959). The pressure on the boundary of the photosphere is of the order of 0.05 atmospheres. The Sun emits a powerful radiation flux the temperature of which is 5,750 K.

Figure 8. Circulation in the Region of the Great Red Spot on Jupiter, Traced on the Basis of Movement of a Small Spot Over the Disc of the Planet. The figures accompanying the dots illustrating the position of the spot are the dates of observation in January 1966. Photograph taken from article by Reese and Smith (1968).

Photographs of the surface of the Sun made with high resolving power show that intensive small-scale convective motions take place here, this being indicated by the cells (granules) of irregular shape, the dimensions of which are of the order of 1000 km along the horizontal and the lifetime of which is around 10 minutes. The depth of the convective zone may be estimated only in theory. Various models of the Sun yield for this /71
depth values ranging from 0.1 to 0.3 of the radius of the star. Frequently, especially during the solar activity maxima, spots are visible on the surface of the Sun — irregularly shaped formations having diameters of several tens of thousands of kilometers, the temperature of which is several hundred degrees below the temperature of the surrounding surface. Owing to the lower temperatures the radiation flux from them is smaller and the spots seem dark in comparison to the remaining regions.

Circulation on the Sun is observed in the form of so-called differential rotation: the angular velocities of rotation on the atmosphere vary from latitude to latitude. The basic material on motions in the atmosphere of the Sun is provided by observations of the spots, on the assumption that the displacements of the latter take place at the speeds of the motions in the atmosphere itself. Individual spots move in a fairly complex manner, but on the average their daily displacement in longitude φ depends on the latitude ϑ in accordance with the law (see de Jager, 1959)

$$\varphi = 14^{\circ}{,}38 - 2^{\circ}{,}96 \sin^2 \vartheta, \tag{16.1}$$

or

$$\varphi = 11^{\circ}{,}42 + 2^{\circ}{,}96 \cos^2 \vartheta. \tag{16.1'}$$

There are no firmly established views regarding the question of the reasons for differential rotation. The first qualitative considerations were advanced by Eddington (1925). According to him, the centrifugal spin forces of the Sun flatten its figure somewhat, and since the release of energy takes place somewhere in the center of the star, the equipotential and isothermic surfaces do not coincide. As a result, the poles should be somewhat hotter than the equatorial regions on one equipotential surface. Hence the conclusion was drawn about the necessity of meridional circulation, which, owing to the Coriolis force, should generate zonal circulation, differential rotation. This greatly resembles the old qualitative theories of general circulation of the Earth's atmosphere (see Lorenz, 1967) which it was necessary to abandon at the end of the 1940s. Astronomers themselves sensed the difficulties at approximately the same time, when Opik (1951) and Cowling (1953) estimated that the speed of such meridional movements is many orders of magnitude lower than 1 cm/sec.

The decisive role of large-scale atmospheric turbulence in maintaining the circulation conditions on Earth (Lorenz, 1967) became clear in the 1950s on the basis of processing of a large body of empirical materials. It was discovered that in the region of the westerly winds the mean angular momentum is transferred along the gradient of angular velocity ω, that is, from regions with lower values of ω to regions with larger values of ω. The greatest stress was placed on this situation by Starr (see his book already referred to), who termed it "negative viscosity."

It has turned out that a similar picture is also observed on the Sun. Ward (1966) (also see Starr, Gillman, 1968; Starr, 1968) processed data of the Greenwich Observatory covering a period of 76 years on the motion of the sunspots /72
and on their basis calculated the rate of transfer of angular momentum. It was found that the angular momentum is transferred toward the equator in both hemispheres, that is, along gradient ω_0. This transfer takes place at such intensity that, if it were suddenly to cease, the solar atmosphere would be set in rotation as a solid only for a few revolutions of the Sun about its axis, that is, for a few months.

One of the basic questions in the negative viscosity concept is that of the source from which energy is drawn by the large-scale vortices themselves which maintain the zonal circulation (differential rotation). Starr (1968) notes the theoretical possibility that the energy comes directly from small-scale convection, so-called "granulation". However, this mechanism is not entirely clear from the physical standpoint, and it is even more difficult to estimate its effectiveness in quantitative terms. Another possibility realized in the atmospheres of the planets lies in the transfer of potential rather than kinetic energy followed by its conversion to kinetic energy.

As has already been pointed out, owing to the influence of the solar spin the mean temperature at the equipotential levels may be different at all latitudes, something which should lead to the occurrence of large-scale motions ultimately transferring heat to the cooler latitudes. These circumstances make it possible to speak of the similarity of certain fundamental features of atmospheric circulation on the Sun and the planets. Hence it is of interest to attempt to apply to the Sun our rather general considerations regarding the similarity of atmospheric circulations, which do not require determination of the details of the picture of currents and energy conversions. The author has earlier discussed the subject in somewhat greater detail than is the case here (see Golitsyn, 1972).

The picture of maintenance of stationary circulation on the average on the Sun may be imagined as follows in physical terms. There is supplied to the solar atmosphere a flux of energy equalling specific luminosity $q = 6.3 \cdot 10^{10}$ erg/cm^2·sec. A certain part of this flux is expended in maintaining the kinetic energy of the large-scale motions. One of the chief aims is to estimate what this portion is.

Other necessary parameters include the radius of the star, which at the level of the photosphere equals (de Jager, 1959) $6.96 \cdot 10^{10}$ cm, $\mu \approx 0.6$, $\kappa = 5/3$, $c_p \approx 3.5 \cdot 10^8$ cm^2/sec^2·K, $g \approx 2.7 \cdot 10^4$ cm/sec^2, and $\omega = 2.4 \cdot 10^{-6}$ sec^{-1}. The values of the similarity criteria are in this instance the following: $\Pi_g \approx 5 \cdot 10^{-4}$, $\Pi_\omega \approx 0.14$.

We do not know the depth to which circulation on the Sun extends, and so we cannot estimate the mass of the unit atmosphere column M, and consequently the energy similarity criterion Π_M. On the other hand, it would be highly desirable somehow to determine the order of magnitude of M.

We make the assumption that the circulation affects fairly great depths of the solar atmosphere, so that $\Pi_M \ll 1$. We thereby adopt the hypothesis regarding the similarity of circulation relative to the precise value of this criterion, and then will verify the validity of this assumption on the basis of the results obtained by means of it. /73

If $\Pi_g \ll 1$ and $\Pi_M \ll 1$, formula (10.1) may be used for the total kinetic energy of circulation:

$$E = 2\pi B_0 f_1(\Pi_\omega)\, \sigma^{1/8} c_p^{-1/2} q^{7/8} r^3. \tag{16.2}$$

In the case of the planetary atmospheres $f_1(\Pi_\omega) = 1$, if $\Pi_\omega \ll 1$. For the Sun $\Pi_\omega = 0.14$, but the situation is radically different here. In the atmospheres of the planets, motions are caused by the uneven heating of the atmospheres by an external source represented by solar radiation. On the Sun the source of energy is an internal one and the large-scale hydrothermodynamic imbalance is universally acknowledged to be caused by the spin of the star (although opinions differ as to the specific mechanisms of creation and occurrence of this imbalance). Rotation is manifested in the existence of centrifugal forces which determine the distribution of matter. Hence it is to be assumed that for the Sun and the stars function $f_1(\Pi_\omega)$ at small values of Π_ω must be proportional to Π_ω^2, that is, ω^2. The constant term must equal zero, unlike the case of the planets, since in a fixed star the distribution of sources of energy and all thermodynamic quantities will be centrally symmetric and large-scale motions cannot arise. As a result

$$f_1(\Pi_M) = a\Pi_M^{1/2} \text{ when } \Pi_M \ll 1. \qquad (16.3)$$

in which a is a constant probably of the order of unity. By analogy with the dense atmosphere of Venus it may be assumed that for the Sun utilization factor k of formula (5.4) is near unity. To be definite we shall assume $ak^{1/2} = 1$. Then, (10.1) and (16.3) being taken into account, formula (16.2) is rewritten in the form

$$E = 2\pi\Pi_M^{1/2}\sigma^{1/4}c_p^{-1/4}q^{3/4}r^3 = 2\pi\sigma^{1/4}c_p^{1/4}q^{1/4}\omega^2 r^4. \qquad (16.4)$$

We multiply additionally and divide this expression by M. Then considering that $4\pi r^2 M = M_0$ is the mass of the atmosphere affected by the motions, taking (6.9) into account we obtain

$$E = \frac{1}{2}M_0\omega^2 r^2\Pi_M. \qquad (16.5)$$

This formula shows that the ratio of the kinetic energy of differential rotation to the total kinetic energy of the rotating layer equals the value of the energy similarity criterion, Π_M.

Formula (16.4) permits estimation of the total kinetic energy of circulation of the solar atmosphere. Using the values of the parameters indicated above, we find that $E = 1\cdot10^{36}$ erg.

On the other hand, as in the case of Jupiter and Saturn (see Section 15), the kinetic energy of apparent differential rotation may be estimated on the basis of observational data, that is, on the basis of formulas (16.1) or (16.1'), as /74

$$E = 4\pi r^2 M\cdot\frac{1}{2}\int_{-\pi/2}^{\pi/2} v^2(\vartheta)\sin^2\vartheta\, d\vartheta, \qquad (16.6)$$

in which $v(\vartheta) = r\sin\vartheta\cdot\Delta\omega$ is the (continuous) deviation from the speed of rotation of the polar regions, which in accordance with (16.1') equals

$$\Delta\omega = \frac{2^\circ{,}96\cos^2\vartheta\cdot 2\pi}{360^\circ\cdot 8{,}64\cdot 10^4 c} = 0{,}6\cdot10^{-6}\cos^2\vartheta \text{ sec}^{-1}$$

Inserting these expressions into (16.6), we find that $E = 10^8$ M erg. Comparing this estimate with the foregoing one, we find $M_0 = 10^{28}$ g. The total mass of the Sun is $M_0 = 2\cdot10^{33}$ g, that is, less than a thousandth of a percent of the total mass of the Sun takes part in the differential rotation, if the latter is assumed to be uniform in depth. Hence, $M = M_0/4\pi r^2 = 2\cdot10^5$ g/cm^2; consequently Π_M, in accordance with (6.19), equals $7\cdot10^{-3} \approx 10^{-2}$, that is, the assumption that its value is small proves to be correct.

The value found for M corresponds to a pressure of $p = Mg = 5.5\cdot10^9$ dyne/cm^2 = 5,500 atmospheres. The depth of the atmosphere affected by motion may also be estimated. On the assumption of adiabaticity, depth z is estimated as

$$z = -\frac{T_e}{\gamma_a}\left[\left(\frac{p}{p_e}\right)^{\frac{\kappa-1}{\kappa}} - 1\right], \tag{16.7}$$

in which γ_a = 8 K/km, the adiabatic temperature gradient; $p_c \approx 0.05$ at, the pressure at the boundary of the photosphere, the radiation temperature of which is $T_e \approx 5750$ K (de Jager, 1959). By means of (16.7) we obtain $z = 7 \cdot 10^4$ km, that is, the atmosphere is in motion to a depth on the order of 0.1 of the solar radius. If it is borne in mind that the intensity of differential rotation may weaken with depth (see, for example, Iroshnikov, 1969), the depth is found to be great. This is in agreement with estimates of the depth of penetration of the convective zone, which, according to Kuiper (1953), is of the order of 10-30% of the solar radius.

Let us now turn to estimates of the other characteristics of general circulation. For the sake of definiteness, as with Earth and Mars, we assume that $\beta/2 \approx 1$. Then the circulation lifetime is $\tau_U \approx r/U$. The mean velocity value, $U = (2E/M_0)^{1/2}$, in accordance with the estimates of E and M_0 obtained in the foregoing, equals 140 m/sec. Hence $\tau_U \approx 5.2 \cdot 10^6$ sec = 60 days. This estimate is in agreement with the results of Ford (1966) cited at the beginning of the section.

Once the value of τ_U is known, it is possible to estimate the mean rate of dissipation (generation) of kinetic energy over the entire atmosphere $\epsilon = E/\tau_U$ = / 75
$= 10^{36}$ erg/$5 \cdot 10^6$ sec $= 2 \cdot 10^{29}$ erg/sec. Hence we have $\varepsilon = \epsilon/M_0 = 20$ cm^2/sec^3 per unit mass. This value applies to large-scale motions, while ε should be substantially larger in small-scale convection manifested in granulation, since there is in operation there another and much more effective mechanism of conversion of potential energy to kinetic: the vertical instability of the atmosphere, to which the energy flow is supplied from below.

Now that we have an estimate of ϵ and know the total energy flux emitted by the Sun, $Q = 4\pi r^2 q$, we estimate the value of η, the efficiency of the atmosphere in converting the total power supplied to it Q to the kinetic energy of large-scale motions. For the Sun $\eta = \epsilon/Q \approx 5 \cdot 10^{-5}$, a value 2.5 orders of magnitude smaller than for the atmosphere of Earth.

The horizontal temperature difference responsible for the general circulation may be estimated, in accordance with formula (5.3) and $\delta T = \eta T_1/k$. According to the adiabatic model, at a depth of 70,000 km $T \approx 5 \cdot 10^5$ K. For k = 0.1-1 we obtain $\delta T \approx$ 250-25 K. As has already been pointed out, it is possible that k is near unity, so that the smaller value of δT appears to be more probable. Thus in the interior of the Sun, where the temperature is around one-half million degrees, the temperature difference on the equipotential surface apparently is only a few dozen degrees.

It should be noted that we do not predict the sign of δT; hence it is impossible to say whether the polar or the equatorial regions of the Sun are warmer, although the concept of somewhat hotter polar regions is more understandable from the physical viewpoint. Direct temperature measurements at the edges of the solar disc at the poles and at the equator yield no definite answer, demonstrating dispersion in the measured temperature values for various observations over the range $\pm(10\text{-}20)$K; this range is much larger than the measurement error. It is believed that the probable cause of dispersion of the measurement data is represented by turbulence, that is, temperature fluctuations are observed which are caused by turbulent convection. In this context we may note that within the limits of the granule, the elementary convective cell, temperature differences of the order of 100 K are observed (de Jager, 1959). Hence protracted systematic observations are required in order to single out the constant temperature difference. We may also note that temperature difference δT by no means need remain constant up to the visible surface of the Sun. Experience with the Earth's atmosphere shows, for example, that the temperatures along the meridian are largely smoothed out at a great distance from the surface of the planet.

If along with the mechanism considered of conversion of the energy supplied to the atmosphere to kinetic energy, one associated with the presence of a weak meridional temperature gradient, there are some other mechanisms in operation, the estimate obtained for δT must be reduced (if, of course, our estimate $E \approx 10^{36}$ erg is correct).

The magnetic fields on the Sun have been disregarded in making all these estimates. If the Sun has a general magnetic field, owing to the hydromagnetic interaction it should decelerate the motion, and then the value of E obtained /76
here should be regarded as the maximum estimate. Hence the estimates of the depths of penetration, efficiency η, and δT are also found to be maximum ones. The generation of local magnetic fields also requires the expenditure of energy, this again reducing the intensity of circulation, to say nothing of the local hydromagnetic interactions. The latter, as is noted by Starr (1968), are what probably represent the basic mechanism which in the absence of a solid surface in the interior of the Sun brings about the balance of the general momentum in circulation. However, it is as yet difficult to make any quantitative estimates of all these effects.

CHAPTER 4. BOUNDARY LAYERS AND TURBULENCE IN THE ATMOSPHERES OF THE PLANETS OF THE EARTH'S GROUP /77

17. Boundary Layers on Mars and Venus

In considering the atmospheric dynamics (with the exception of the first half of section 11) we have up to this point not taken the interaction between the atmosphere and the underlying surface into account. This interaction takes place through the boundary layer, in which the speed drops sharply to zero and the temperature strives toward the surface temperature. Exchange of heat, momentum, and moment of momentum between the atmosphere and the solid substance of the planet take place through the boundary layer. Knowledge of the vertical wind profiles in the boundary layers is necessary also for purely practical purposes, to accomplish a soft landing on the surface of other planets. We shall restrict consideration here to the planets nearest Earth, Mars and Venus, since even if the large planets have a solid surface, it must be situated at a very great depth.

The theory of the boundary layer of the atmosphere has been fairly fully elaborated, the basic advances made in it being due to consistent application of considerations of similarity and dimensionality. The contemporary theory of the atmospheric boundary layer originated in the work of Obukhov (1946) and Monin-Obukhov (1953, 1954). A detailed account of the theory, together with a survey of experimental data, is to be found in the book by Monin and Yaglom (1965, Chapter 4), as well as in the book by Zilitinkevich (1970) devoted expressly to this one question. We will keep precisely these works in mind subsequently in our brief presentation of the main theoretical concepts, without making special mention of this circumstance in each instance. Under ground conditions, both in the atmosphere and in wind tunnels, a large body of empirical material has been assembled which confirms the conclusions of the theory.

Three different component layers, the processes in which are determined by various factors, may be distinguished in the boundary layer of the atmosphere. Firstly there is the very thin layer immediately adjacent to the surface of the ground, in which molecular exchange processes are of vital importance. The thickness of this layer is determined by the thickness of the viscous sublayer or by the mean height of the surface roughness. The most important consequence /78
of the existence of this layer is the fact that there may be appreciable temperature discontinuities in it. We shall later return to them and consider them in detail.

The next layer, and the most thoroughly studied one, is the ground layer, in which there is approximate consistancy of the turbulent flows of momentum $\tau = -\rho\overline{u'w'}$ and heat $q_h = c_p\rho\overline{w'T'}$, in which u', w', and T' are pulsations of the horizontal and vertical speed and temperature components, and ρ is the density of the atmosphere. Under terrestrial conditions its thickness is of the order of several dozen meters. The chief abrupt variations in the vertical wind and temperature profiles are observed in this layer. Above it to an altitude of approximately 1 km in the Earth's atmosphere, there is distinguished the planetary boundary layer, or as it is otherwise termed the Ekman layer, in which the temperature and wind speed modulus change very little in comparison with the ground layer but the wind speed vector turns with altitude owing to variation in the balance among the pressure gradient, the Coriolis acceleration, and the viscous terms (Reynolds stresses).

For the Earth's atmosphere the basic direction of research is toward obtaining estimates of the turbulent flows of momentum and heat on the basis of data of measurements of the profiles of mean speed $u(z)$ and temperature $T(z)$ for the ground layer or on the basis of data on the speed of the geostrophic wind U_g and on the potential temperature drop $\delta\theta$ for the planetary boundary layer of the atmosphere[11]. For the other planets it is of interest to obtain at least rough estimates of the trace of the mean speed and temperature profiles in their boundary layers. We have the necessary data at our disposal for this purpose, possessing above all estimates of the mean wind speeds U obtained in the previous chapter.

Let us first consider the ground layer. Once mean speed U is known, we can estimate the dynamic speed, which is also termed the frictional speed, $U_* = \sqrt{\tau/\rho}$. In the Earth's atmosphere the quantity $u_*/U \approx$ 2-5%, depending on its temperature stratification (the first figure refers to high stability, the rise in potential temperature with altitude usually observed at night, and the second to great instability, convection taking place during the day in summer). There is an obvious upper restriction on the value of the second parameter, turbulent thermal flux q_t: it may not exceed q, the value of the energy flux per unit surface. Even under conditions of advanced convection ratio $q_t/q \approx 0.1$ for Earth. In the case of stable stratification, when the atmosphere is warmer than the Earth's surface, $q_t < 0$, that is, the thermal fluxes are directed toward the ground, and the modulus of ratio q_t/q is usually several times smaller or even smaller by an order of magnitude than during the day. In the atmospheres of Mars and Venus there is no reason to expect too substantial a departure from /79
the patterns observed in the Earth's atmosphere. In addition, it is possible to make a qualitative estimate of the direction in which such departures might act on these planets.

Thus in a certain sense we are faced with a problem opposite that on Earth: having some idea of the turbulent flows of momentum and heat, we must estimate the thickness of the boundary layer and determine the trace of the mean speed and temperature profiles.

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[11]The potential temperature is related to the ordinary temperature by the relation $d\theta(z)/dz = dT(z)/dz + \gamma_a$. If the entropy of the atmosphere were constant, this corresponding to adiabatic mixing of the atmosphere, when $dT/dz = -\gamma_a$, the potential temperature would remain a constant value.

According to the general theory the structure of turbulence and the trace of the mean profiles in the temperature stratified ground layer are defined by the following parameters: $q' = q_t/c_p\rho$, the normalized turbulent heat flux; $u_* = \sqrt{\tau/\rho}$, the dynamic velocity; and buoyancy parameter $g\beta$, where g is the acceleration of gravity and β is the dilation, which for an ideal gas equals T_0^{-1}, where T_0 is the characteristic temperature of the medium. From these parameters we can construct length scale

$$L = -\frac{u_*^3}{\varkappa g\beta q'}, \tag{17.1}$$

which is usually termed the Monin-Obukhov scale, and temperature scale

$$T_* = \frac{q'}{\varkappa u_*}, \tag{17.2}$$

in which κ is the Karman constant. The vertical profiles of the mean speed and potential temperature are universal functions of dimensionless altitude $\zeta = z/L$:

$$u(z) = \frac{u_*}{\varkappa}\left[f_u\left(\frac{z}{L}\right) - f_u\left(\frac{z_0}{L}\right)\right], \tag{17.3}$$

$$\theta(z) = \theta_0 + T_*\left[f_\theta\left(\frac{z}{L}\right) - f_\theta\left(\frac{z_0}{L}\right)\right], \tag{17.4}$$

in which z_0 is the surface roughness height, and θ_0 is the potential temperature value at the level $z = z_0$. For universal functions f_u and f_θ we have the following expressions obtained from the considerations of similarity and dimensionality:

$$f_u(\zeta) = f_\theta(\zeta) \quad \begin{cases} \ln\zeta + \beta\zeta, & 0 < \zeta, \\ \ln|\zeta| + \beta'\zeta, & \zeta_1 \leq \zeta < 0, \\ a + C\zeta^{-1/3}, & \zeta < \zeta_1. \end{cases} \tag{17.5}$$

According to the thorough statistical treatment of the extensive empirical material made by Zilitinkevich and Chalikov (1968) (also see Zilitinkevich, 1970): $\kappa = 0.43$; $\beta = 9.9$; $\beta' = 1.45$; $\zeta_1 = -0.16$; $a = 0.24$; and $C = 1.25$.

Formulas (17.3)-(17.5) are valid for the ground layer, in which the variation in turbulent fluxes τ and q_t with altitude may be disregarded. Precisely from this viewpoint do Monin and Obukhov (1954) give the following estimate of the ground layer thickness: /80

$$H_n < \frac{a u_*^2(0)}{l U_g}, \tag{17.6}$$

in which $\alpha = \dfrac{u_*^2(0) - u_*^2(H)}{u_*^2(0)}$ is the relative variation in frictional stress τ, l is the Coriolis parameter, and U_g is the geostrophic wind speed, that is, the wind speed in the free atmosphere. For the Earth's atmosphere we obtain H = 50 M when α = 20% and u_*/U = 5%. For Mars we obtain H ≈ 100-150 M at the same value of α, since the Coriolis parameter has virtually the same value but the mean wind speeds are 2-3 times higher than those on Earth. For slowly rotating Venus and for the equatorial regions of Earth and Mars, where Coriolis parameter $l = 2\omega\sin\vartheta$ is small, we may adopt the altitude at which the wind speed is comparable to the speed in the free atmosphere as the thickness of the ground layer, or more precisely the boundary layer. As we shall see later, this thickness usually is of the order of several units on the Monin-Obukhov scale.

Let us consider briefly the planetary boundary layer, which may be determined for Earth and Mars. The action of the Coriolis force is substantial in this case. The thickness of this layer may be defined as

$$L_* = \frac{\varkappa u_*}{l}. \tag{17.7}$$

For Earth $L_* \approx 1$ km, while for Mars it is 2-3 times as great. The turning angle of the wind with altitude depends on dimensionless stratification parameter $\mu = L_*/L = \kappa^3\beta T_*/lu_*$. Under terrestrial conditions the total turning angle of the wind with altitude is of the order of several degrees in the case of convection and reaches approximately 40° at high stability. Similar figures are to be anticipated for Mars as well.

Let us return to the ground layer. Formulas (17.3) and (17.4), with (17.5) taken into account, include a parameter unknown for the other planets, the height of the dynamic surface roughness of the planet, z_0. Fortunately, it is included as a logarithm, and so even an approximate estimate of its order of magnitude is sufficient for our purposes. According to Table 1.1 in the book by Zilitinkevich (1970), under terrestrial conditions on the average $z_0 \approx 1$ for land, $z_0 \approx 0.01$-0.1 cm for deserts, and even $z_0 \approx 1$ M for forests. In view of the fact that the surfaces of Mars and Venus resemble a desert more than anything and that the surface of Mars may be fairly uneven, we shall adopt $z_0 \approx 1$ cm, although values an order of magnitude smaller are also probable.

Once the speed and temperature profiles are known, it it possible to determine the parameter of local hydrostatic stability of the atmosphere: the Richardson number, the ratio of the convective and dynamic factors . the /81
generation of turbulence

$$\mathrm{Ri} = g\beta\frac{d\theta/dz}{(du/dz)^2} = \zeta\varphi(\zeta). \tag{17.8}$$

in which universal function $\varphi(z)$ is defined as

$$\varphi(\zeta) = \frac{\varkappa z}{u_*}\frac{du}{dz} = \frac{z}{T_*}\frac{d\theta}{dz}. \tag{17.9}$$

It is assumed in this instance that the turbulent exchange coefficients for momentum K and heat K_t introduced in accordance with the equations

$$\tau = \rho K\frac{du}{dz}, \quad q_T = -\rho c_p K_T\frac{d\theta}{dz},$$

are the same. We may note that this is known not to be the case for high stability, and then factor $\alpha = K_t/K$, the inverse turbulent Prandtl number, which becomes less than unity for the case of stability, is to be inserted in the denominator of the righthand member of equation (17.8). For high stability this factor is larger than unity, fairly rapidly approaching 3 or even 4 (Zilitinkevich, 1970). The universal functions for velocity and temperature will also differ by the amount of this factor ($f_u = \alpha f_\theta$). We will not take this effect into account here because of the great indefiniteness of a number of other factors and the approximate nature of our estimates.

The coefficient of turbulent mixing $K = \kappa u_* L Ri$ in the ground layer is expressed by the following formulas:

$$K = \varkappa u_* z, \; |L| \to \infty, \tag{17.10}$$

$$K = \frac{\varkappa u_* z}{1 + 9z/L}, \; |L| < \infty, \tag{17.11}$$

$$K = \frac{3u_* z}{C}\left(\frac{z}{L}\right)^{1/3}, \; \zeta = \frac{z}{L} < \zeta_1. \tag{17.12}$$

The first of these equations applies to neutral stratification observed during the morning and evening hours, when the temperature profile is near the adiabatic, $|q_t| \to 0$, and the forces of buoyancy are insignificant, the second equation to conditions of stability and slight instability, and the third to convection.

In Table 8 there are given for Mars, Venus, and Earth (for purposes of comparison) the values of buoyancy parameter $g\beta$, dynamic velocity u_* (which equals 3% of the mean wind speed over the atmosphere), the normalized turbulent thermal flux $|q_t'|$ for free convection conditions ($q_t/q = 0.1$), and the corresponding values of temperature scale T_* and the Monin-Obukhov length L.

For Mars the conditions adopted were those at the equator during the period of solstice, as calculated by Leovy and Mintz (1969), where U = 20 m/sec. These conditions are the maximum ones as regards the value of the flux of solar heat arriving at the surface. Owing to the wide variation in wind speeds /82
on the planet, great variation is also to be expected in the values of dynamic velocity u_*, and accordingly in values T_* and $|L|$ as well.

The data of Table 8 show that the basic parameters determining the structure of the ground layer, the dynamic velocity and especially temperature scale T, vary substantially for all 3 planets because of the great difference in the basic atmospheric parameters and above all in density. Hence the ground layer on each planet must have distinct features of its own, and we shall now proceed to consider them.

TABLE 8. GROUND LAYER PARAMETERS FOR CONVECTION CONDITIONS.

Planet	gp cm (K·sec	u_* cm/sec	q_T' K·cm/sec	T_* K	L m
Mars	1,7	100	600	11	20
Venus	1,2	3	0,08	0,01	900
Earth	3,3	50	7	0,3	100

Commas indicate decimal points.

Mars

The dynamic and especially the thermal structure of the lower part of the Martian atmosphere has been examined in fairly great detail by Gierasch and Goody (1968). They have performed numerical calculations of the vertical temperature profiles and convection conditions for a model of the atmosphere with p_s = 5 mb, for different latitudes, seasons, and times of the day. An estimate of the mean wind speed of U = 40 m/sec was obtained with the formula derived by them, which essentially coincides with the thermal wind formula. However, the vertical mean wind profiles could not be found in the model considered by them, and the authors confined themselves to making a very rough estimate of the Richardson numbers for various conditions. We may note that their numerical estimate of quantity q_t at noon on the equator is of the same order as ours (q_t = 0.1 q).

Under convection conditions the "logarithmic plus linear law" for wind and temperature profiles (17.5) may be adopted up to values ζ_1 = -0.16, that is, at L = -20 M up to an altitude of 3.2 and above the surface of the planet. In this instance (z_0 = 1 cm)

$$u(z) \quad 2{,}3\left|\ln(100z) \cdot \frac{z}{14}\right|,$$

$$\theta(z) \approx T(z) \quad \Gamma_0 \; . \; 14\left|\ln(100z) \quad \frac{z}{14}\right|,$$

in which u(z) is expressed in m/sec and z in meters. At an altitude of 3.2 m u = 13 m/sec, and ΔT = T(0) - T(3.2) ≈ 70 K. Thus over a layer of the atmosphere only 3 m deep the velocity reaches almost one-half of its value typical of the free atmosphere, and the temperature discontinuity (in this instance the difference between usual temperature T and potential temperature $\theta = T + \gamma_a z$, in which for Mars γ_a ≈ 5 K/km plays no role whatever) reaches 70 K. Abrupt variations, although smaller ones, in temperature during the day in the lowest layer of the atmosphere have been found by Gierasch and Goody (1968). We may note that they were unable to obtain such abrupt variations as in our case, since they assumed the vertical step in numerical calculation to equal 100 m. /83

The possibility of existence of abrupt temperature variations was also pointed out by Gifford as early as 1956 (also see Moroz, 1967, Sections 2, 4), who cited the example of temperature measurements in summer at noon in the

Gobi Desert. According to these measurements, the temperature difference between the surface of the ground and the air at a height of 2 m reaches 20 K. However, the inequality of heat and momentum exchange comes into play in the case of great instability: the inverse turbulent Prandtl number α in this instance tends (when $z/L \approx 2$) toward 3, this accordingly reducing the abruptness of temperature variations. The increase in α takes place fairly rapidly, and even at $\zeta = -0.2$, that is, where in our case $z \approx 3$m, $\alpha \approx 2$, and then the temperature discontinuity will be 35 K rather than 70 K. For altitudes greater than 3 m, it is necessary to use the last of formulas (17.5), which describes 3 convection conditions. In this instance the mean speed asymptotically approaches its maximum wind speed value in the free atmosphere. When $|\zeta| \approx 5$, that is, $z = 100$ m, the speed reaches approximately 90% of its value at $|\zeta| \to \infty$. The coefficient of turbulent exchange increases rapidly with altitude. For $z = 100$ m, we obtain $K = 4 \cdot 10^6$ cm^2/sec in accordance with (17.12), a value which may be contrasted with the estimate by Gierasch and Goody (1968), $K = 10^8$ cm^2/sec, for the lower kilometer layer obtained numerically under the same conditions of equatorial noon, since $K \sim z^{4/3}$ and $10^{4/3} \approx 20$.

In the case of stable stratification (nighttime) we adopt $L = 150$ m, $T_* = 2$ K. Then a speed of 20 m/sec will be reached at an altitude of around 120 m, and the temperature drop will in this instance be of the order of 40 K. The coefficient of turbulent exchange K calculated from (17.11) will be of the order of 40 K. Coefficient of turbulent exchange K calculated with (17.11) will be of the order of 10^5 cm^2/sec. According to Gierasch and Goody (1968), the effects of radiation attentuation of temperature fluctuations should be essential in establishment of the temperature conditions at this value of K (see Goody, 1964; Golitsyn, 1963, 1964). This should lead to decrease in q_t' and T_*, that is, to increase in L and thus to decrease in the temperature drop for the assigned altitude range.

The Coriolis force must be taken into account above the 100 m level, that is, the planetary boundary layer referred to earlier has its beginning here.

Let us consider yet another question relating to the temperature discontinuity in the molecular sublayer directly adjoining the surface itself. This question has been investigated in fairly great detail in engineering, but apparently Zilitinkevich (1970) was the first to call attention to its importance in meteorology. The considerations of similarity established that this discontinuity should be a function of the Reynolds number. According to the empirical data presented in his book, the value of this discontinuity at a Prandtl number of the order of unity may be described by the formula /84

$$\delta\theta = 0.27\, T_*\, \mathrm{Re}_0^{0.45}, \qquad (17.13)$$

in which $\mathrm{Re}_0 = 30\, u_* z_0/\nu$ is the Reynolds number for this sublayer. For the rarefied atmosphere of Mars $\nu \approx 10$ cm^2/sec, and if $z_0 = 1$ cm, then $\mathrm{Re}_0 = 300$. Then according to (17.13) the temperature discontinuity at noon on the equator equals 30 K. If $z_0 = 0.1$ cm, then $\delta\theta = 10$ K. Hence, wh. at noon on the equatoı the surface temperature reaches values of $T = \sqrt{2T_e} = 310$ K, at an altitude of 3 m it may be many dozen degrees lower.

All this demonstrates that the micrometeorology of the equatorial regions must be highly unusual. In the temperate latitudes, especially in the winter hemisphere, the values of q_t, and consequently of T_* as well, should be appreciably smaller and the temperature variations should not be so abrupt.

Venus

The values given in Table 7 for the boundary layer parameters q_t', T_*, and L for the atmosphere of Venus should be regarded as limiting values, T_* being a maximum and L a minimum, since they were calculated on the assumption that the entire flux of direct solar radiation q reaches the surface of the planet. In this instance $T_* \sim \gamma q$, and $L \sim (\gamma q)^{-1}$, in which γ is the portion of the radiation flux reaching the surface. We note immediately that if the value of γ is small, then $L \to \infty$, th[illegible] corresponding to purely neutral stratification, that is, the velocity profil[illegible] be logarithmic, and the potential temperature constant with altitude, since $T_* \to 0$, that is, $T(z) = T_0 - \gamma_a z$.

Let us first estimate the altitude at which the speed calculated from the formula $u(z) = (u_*/\kappa)\ln(z/z_0)$ is comparable to the mean speed of 5 m/sec. This altitude is estimated on the basis of the formula $z = z_0\exp(\kappa u/u_*)$. Hence we obtain an altitude of around 1 km when $z_0 = 1$ cm and $u_*/U = 0.05$ (a figure corresponding to neutral stratification). However, this estimate is highly sensitive both to the value of z_0 and especially to u_*, and so it should be regarded expressly as an estimate of the order of magnitude.

Let us consider the limiting case to which the values of u_*, T_*, and -L given in Table 7 correspond. If -L = 900 m, then under convection conditions the speed will reach its limiting value in the free atmosphere at $|\zeta| \approx 5$, that is, $z \approx 4.5$ km. However, the departure of the temperature from the adiabatic profile does not exceed 1°, owing to the small value of T_*. Such departures should be even smaller at night.

The coefficient of vertical turbulent exchange at an altitude of 1 km will be $K \approx \kappa u_{*}z \approx 5 \cdot 10^5$ cm^2/sec.

18. Turbulence in the Free Atmosphere /85

Quantity ε, the rate of dissipation of kinetic energy, serves as the basic characteristic of turbulence, and specifically of fluctuations in speed and turbulent mixing. In the two preceding chapters general formulas were proposed and estimates were given of ε, so to speak, on the global scale, on the average for the entire atmosphere. The example of Earth's atmosphere shows that dissipation is distributed very unevenly in altitude: the lower kilometer boundary layer, according to contemporary estimates (Zilitinkevich, 1970; Lorenz, 1967) accounts for one-half to two-thirds of all dissipation. Hence in the boundary layer, especially in the lower part of it, the ground layer, the value of ε may be substantially larger, but in the bulk of the atmosphere appreciably smaller than the value of our global estimate of ε. The nature of variation in quantity ε as a function of stratification and altitude has been described, for example, in the book by Monin and Yaglom (1965, Chapter 4). We may note that in the case of highly unstable stratification, ε is constant with altitude in the convection layer, and decreases with altitude as z^{-1} in the case of stratification near the neutral and stable stratification. However, since such basic characteristics of turbulence as the root mean square difference in speed fluctuations at two points and the coefficient of turbulent mixing are proportional to $\varepsilon^{1/3}$, in the first approximation the vertical distribution of ε should not play a very significant part.

For Earth $\varepsilon = 4$ cm^2/sec^3, for Mars, according to Leovy and Mintz (1969), $\varepsilon = 10$-20 cm^2/sec^3, and for Venus, according to the formulas in Section 8, $\varepsilon \approx 10^{-2}$ cm^2/sec^3. Hence on Mars the speed fluctuations and the mixing are on the average 1.5 times more intensive than on Earth, but on Venus are an order of magnitude smaller.

Also of practical interest are estimates of the temperature fluctuations in the atmospheres of the planets, since these fluctuations cause fluctuations in the radiowave refraction index. Thus a radio signal passing through the atmosphere of a planet will undergo fluctuations in amplitude and phase.

To determine the intensity of temperature fluctuations, Obukhov (1949a) introduced the mean rate of smoothing of the temperature field heterogeneity N, defined as

$$N = K\overline{\left(\frac{\partial T}{\partial x_i}\right)^2} \tag{18.1}$$

on the analogy of the Rayleigh definition of dissipation rate ε (see Landau, Lifshits, 1954). Hence quantity N is often termed simply the temperature dissipation. For slowly rotating planets $K \approx 0.1$ Ur and $\partial T/\partial x_i \sim \delta T/r$. Thus the value of N may be estimated by means of the formulas of Section 8.

The intensity of the temperature fluctuations is determined by structural constant c_t^2, introduced by the equation

$$D_t(a) = \overline{(\Delta_a T)^2} = C_2^2 N \varepsilon^{-1/3} a^{2/3} = C_T^2 a^{2/3}, \tag{18.2}$$

in which $D_t(a)$ is the structural function of temperature, that is, the root /86
mean square temperature difference for two points separated by distance a; $C_2^2 \approx$ ≈ 3 (see Monin, Yagolom, 1967, Section 23). Once N and ε are known, it is possible to determine C_t^2.

Refraction index n for $|n - 1| \ll 1$ is of the form $n = 1 + \beta_1 p/T$, in which β_1 is a constant determined by the composition. For air $\beta_1 = 0.08$ K/at, and for CO_2 $\beta_1 = 0.13$ K/at. Fluctuations in the refraction index are defined as $\delta n = -\beta_1 p \delta T/T^2$, since the pressure fluctuations are insignificant (see Tatarskiy, 1967). If the temperature fluctuations are described by formula (18.2), the analogous formula will be correct for the refraction index fluctuations as well. For this reason the corresponding structural characteristic C_n^2 may also be estimated by means of the external parameters (Golitsyn, 1970b). In this instance an estimate correct in order of magnitude is obtained for Earth's atmosphere. This moved the author, working in collaboration with Gurvich (1971), to apply similar considerations to estimation of the fluctuations in the amplitude of the signal from Mariner-5 of the wavelength of 13 cm which passed through the atmosphere of Venus, as it emerged from behind the disc of the planet. Calculations yielded a value for the fluctuations in the signal level of the order observed in measurements. Similar values of fluctuation intensity were also observed on the occultation of Mariner-5 (Kliore et al., 1967), as has been demonstrated by Gurvich (1969).

19. Dust Storms on Mars

When in November 1971 Mars was approached by the unmanned space stations Mariner-9 and then by Mars-2 and Mars-3, it was found that the entire planet was covered by an unbroken cloud of dust through which only a few of the highest mountains and craters could be indistinctly seen. In Figures 3 and 4 we have already presented photographs of individual regions of the planet taken during the dust storm.

Everyone naturally associates the storm with winds, and so it is the duty of meteorologists and specialists in atmospheric physics to answer the question of what such a dust storm is, how it may be generated and what fosters or impedes it, as well as how it develops and why it ultimately dies out. This is an unusually complex phenomenon, and elaboration of a theory of the dust storms will take many years. To attract the attention of investigators of the atmosphere, in this concluding section of the book a brief survey will be presented of the results of observations of the dust storms on Mars and certain qualitative considerations will be advanced (see Golitsyn, 1973) regarding the probable mechanism of generation, development, and fading of these storms. These considerations are in many respects based on material already presented in this book (see, for example, Sections 13, 17, and 18), this serving as additional justification for discussion of this matter.

A description of the development of the 1971 dust storm on the basis of the data of ground astronomic observations of Mars has been made by Capen and Martin (1972), and on the basis of the data of Mariner-9 by Leovy et al. (1972); also see the popular science article by the author, "Dust Storms on Mars," in the collection Chelovek i Stikhiya (Man and Elements) (Gidrometeoizdat, 1973). /87

Dust storms of high intensity on Mars are known only for the times of great oppositions, when Mars is subjected to intensive observations and the storms hamper such observations. It is known (Glasstone, 1968) that during the great oppositions Mars is near perihelion, and at this time it is the end of spring and beginning of summer in its southern hemisphere. The insolation is at its maximum in this instance, being 20% higher than the average owing to the elongation of the orbit. As long ago as 1909, Antoniadi (see Glasstone, 1968) advanced the idea that, although the yellow clouds presumably consisting of dust particles are also encountered at other times, the most intensive clouds must develop during the perihelial oppositions.

Data on the yellow clouds observed on Mars have been assembled by Gifford (1964). They indicate that the most intensive ones have as a matter of fact been observed during the major oppositions of 1892, 1924, and 1956. We may add 1971 to this list.

However, not every major opposition is accompanied by storms. Thus Gifford's list does not include clouds or the oppositions of 1909 and 1939, although large clouds were observed for the oppositions of 1907 1911, and 1941, when Mars was not too far from its perihelion.

Consecutive major oppositions of Mars are separated by a period of 15 or 17 years. Over this period Mars is near its perihelion another 7 or 8 times. However, at this time Mars either is too far from Earth or is in the daytime sky, that is, astronomical observation of it is impossible. Hence the association of the storms with the major oppositions is the result of sampling the observations.

Thus it appears that the presence of Mars in the vicinity of perihelion is a necessary but far from sufficient condition for the origination and development of a storm. The varying duration, intensity, and space scale of the different storms indicate the importance of local and general meteorological conditions in the Martian atmosphere during the origination and development of a storm. However, the fact that the storms often cover virtually the entire planet indicates that there are certain feedback mechanisms favoring global distribution of the dust as soon as the dust cloud has assumed a sufficiently large size. Unfortunately, we as yet know too little about Mars and are well aware that the structure of the Martian winds must be highly diversified owing to the complexity of the topography, and that description of the global dust storm – terrestrial meteorology has no knowledge of similar problems – requires allowance for too many factors, including ones as yet unknown or poorly understood and little studied, and the complex and often /88
very obscure interaction of such factors. Among these factors consideration must obviously be given to the questions of elevation of the dust into the atmosphere and the wind structure in the surface layer of the atmosphere, the spread of the dust in the atmosphere, absorption of solar radiation by the dust and resulting modification of the temperature conditions in the atmosphere, something in turn leading to change in the wind, and so forth. Thus it would seem that in this difficult situation even simple qualitative considerations regarding the individual stages of a storm might be of some use.

Let us deal first of all with the questions of blowing of the dust from the surface and its elevation into the air. Two qualitatively different mechanisms of this process are known (they are discussed for Martian conditions, for example, in the works of Gifford (1964), Sagan, and Pollack (1969), and Sagan, Veverka, and Gierasch (1971)). The first mechanism is the saltation studied by Bagnold (1941). At a certain wind stress τ depending on the wind speed and its vertical gradient, particles of a certain size a begin to be set in motion: they are lifted into the air to a certain height and are transported by the wind, but then fall under the action of gravity.

The fall of the particles is described by the Stokes-Cunningham equation, which allows for the finite nature of the ratio of the mean free path to particle size (it has been used for Martian conditions by Anderson, 1967).

Calculation of the dependence of the radius of lifted particles on frictional stress τ, or more precisely on frictional velocity $u_* = \sqrt{\tau/\rho}$, has been performed by Sagan and Pollack (1969) for pressures of 5 and 15 mb on the surface of Mars. At $p_s = 5$ mb, when the threshold value is $u_* = 4$ m/sec, particles of a radius of around 200 μ begin to rise. Smaller particles are not listed, since they are entirely imbedded in the viscous sublayer, the thickness of which is μ/u_* (see Landau, Lifshits, 1954; Monin, Yaglom, 1965), and are not affected by the turbulent frictional stress; the flow cannot lift large heavy particles, since the lifting power is proportional to the surface of the particle, and the force of gravity is proportional to their volume. Larger and smaller particles are lifted at values of u_* greater than the threshold value. Thus at $u_* = 6$ m/sec particles of a radius of 50 to 1000 μ are lifted. With increase in pressure the threshold value of u_* decreases, and at $p_s = 15$ mb it is slightly less than 2 m/sec.

Relatively large particles in falling to Earth, may exchange momentum with smaller particles and lift the latter into the atmosphere. As a result of turbulent mixing small slowly falling particles may be spread to greater altitudes. Almost no quantitative study has been made of the effectiveness of this method of introduction of small particles into the atmosphere. At the same time, this is one of the questions of cardinal importance in understanding the origination and development of a storm.

The nature of the behavior of the values of u_* under Martian conditions /89 may be determined if the wind speeds in the free atmosphere are known (see Sections 13, 17). According to Section 17, the value of u_* is determined by wind speed U and is 2-5% of U depending on the stratification: the values of u_* are smaller under conditions of high stability and larger under high instability, free convection. We examined the structure of the ground layer of the Martian atmosphere in Section 17. On the basis of the data presented, and using the typical wind values obtained by Leovy and Mintz (1966) for a model with $p_s = 5$ mb, we can calculate the typical values of u_* in m/sec for winter and summer in the middle latitudes:

Winter	3
Summer:	
morning, evening.....	1
noon	2
midnight.............	1

Since we do not as yet know the mean surface level on Mars, the question arises of how the values of the mean wind for the entire atmosphere vary on change in mean value $\bar{p}_s$. According to Section 10, $U \sim M^{-1/3} = (g/\bar{p}_s)^{1/2}$, in which g is the acceleration of gravity. Hence, other conditions being equal, $u_* \sim \bar{p}_s^{-1/2}$, that is, at the contemporary estimates $\bar{p}_s = 6 \pm 2$ mb (Kliore et al., 1972) the values given for u_* may be regarded as accurate for all values p_s with an error not exceeding 25%.

Comparing the data obtained with the findings of Sagan and Pollack (1969), we see that the values of u_* are in summer 2-4 times smaller than the threshold value of u_*. The winter conditions would seem to be the more favorable ones, but during the cold seasons, as at night, the cohesion between the particles may increase owing to freezing of the moisture, and for this reason the conditions are not very favorable for lifting of dust in the winter hemisphere. This is also confirmed by the data of Gifford (1964), which indicate virtual absence of clouds during the cold season of the year.

Thus lifting of dust should be expected around noon, not under all conditions but in the case of local approximately two-fold excess of u_*, which is here taken as equalling 40 m/sec. Mention may be made of the number of factors favoring the lifting of dust even at lower speeds. First of all there are the turbulent gusts. Their statistical characteristics have been thoroughly studied for the ground layer of the atmosphere. The ratio of the root mean square value of speed pulsations σ_u to u_* (see Zilitinkevich, 1970) equals 2-2.5, this representing 10% of the mean speed. The probability distribution for the speed pulsations is more or less near the normal. This means that increase in speed by 10%, and consequently in u_* as well, is fairly often encountered. Stronger gusts may also occur, although they are less probable[12]. /90

[12]In the work by Hess (1973), who converted the data of Bagnold (1941) on the threshold value of the frictional speed, u_{*th}, for Martian conditions, it is demonstrated that the values of u_{*th} are almost 2 times smaller than those adopted by Sagan and Pollack (1969). Thus at $p_s = 5$ mb, according to Hess, $u_{*th} = 2.5$ m/sec, while the other two authors give 4 m/sec. This greatly increases the probability of commencement of lifting of dust into the atmosphere without requiring overly large wind speed values. The author wishes to express his gratitude to S. Hess for making a preprint of his paper available to him.

Appreciable increase in u_* may occur on abrupt change in the structure of the underlying surface, specifically, on increase in roughness height z_0. This fact has been studied under terrestrial conditions (see Laykhtman, 1970). An approximate analytical solution of the problem has been obtained by Radikevich (1971). The latter has calculated that on tenfold change in z_0 the value of τ increases by approximately 30% at a great distance from the boundary separating regions characterized by different values z_0, while τ undergoes more abrupt change in the vicinity of the boundary[13]. In the region near the dividing line there appear vertical speeds (ascending ones on increase in z_0) which reach several centimeters per second. Field measurements performed under conditions of much more abrupt variation in z_0 also indicate greater variation in τ (an approximately threefold variation; see Shir, 1972).

Another mechanism of transport of dust into the atmosphere is represented by dust storms, the so-called "dust devils." This mechanism has been discussed by Sagan and Pollack (1969) and by Sagan, Veverka, and Gierasch (1971). The conditions of formation of dust devils have been studied by Ryan and Carrol (1970) in the Mojave Desert in southern California. They are formed in a low wind under conditions of great overheating of the ground surface, that is, high instability of the atmosphere. For Mars such conditions are best fulfilled during the noon hours near perihelion. The occurrence of dust devils on Mars must be fostered by the possibility of existence of sharp temperature discontunuities between the ground surface and the atmosphere during the noon hours in the summer hemisphere. As was shown in Section 17, these discontinuities may reach 30 K.

Unfortunately, neither experimental nor theoretical quantitative estimates have been made of the effectiveness of the dust devils in lifting dust. This effectiveness obviously also depends on the amount of dust on the surface which may be lifted into the atmosphere. However, the latter is an areomorphological rather than a meteorological factor. At the present time the possibility is not to be discounted (see Sagan, Veverka, and Gierasch, 1971) that precisely /91
the dust devils may be the chief mechanism placing fine and long-suspended dust in the Martian atmosphere, since in these dust devils the vertical velocities are of the order of the horizontal ones and reach several meters per second under terrestrial conditions.

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[13]The last illustration in the article by Leovy et al. (1972), may be interpreted as confirmation of this effect under Martian conditions. In this figure there are two photographs of the same region of Mars taken at different times from Mariner-9. In one of them the central portion of the region is covered by a diffuse low cloud, a local dust storm. In the second photograph, taken several weeks after the first one, it is clearly to be seen that the part of the surface earlier covered by the storm has a much rougher small-scale topography than the territory surrounding it, which was also uncovered at the [illegible] The author would like to express his gratitude to C. Leovy, who [illegible] reprint of his article and called attention to this photograph.

The foregoing discussion shows that the lifting of any significant quantity of dust into the atmosphere requires simultaneous satisfaction of a number of conditions. The best possibilities are probably provided in the vicinity of perihelion. This makes understandable the rarity of observations of yellow clouds on Mars and of correlation of their appearance with major oppositions.

But let us assume that a dust cloud has assumed sufficiently large dimensions and the concentration of dust in it has become large enough, so that to describe the motion of such a cloud above the underlying surface it is necessary to allow for the opposite effect of the dust on the flow dynamics. The nature of turbulent flows containing a heavy admixture has been studied by Barenblatt (1955). One of the chief results of this study is that in the case of neutral stratification the mean velocity profile of a stationary and horizontally homogeneous flow is modified (see Section 17) and tends with altitude toward the form of

$$u(z) = \frac{u_*}{\varkappa\omega} \ln \frac{z}{z_0}. \tag{19.1}$$

Dimensionless parameter ω is defined as

$$\omega = \frac{v}{\alpha\varkappa u_*}, \tag{19.2}$$

in which v is the particle precipitation velocity, and α is the ratio of the turbulent exchange coefficients for the admixture and the momentum, and is approximately equal to unity. For sufficiently small particles $v < \alpha\kappa u_*$, that is, $\omega < 1$, and then formula (19.1) may be interpreted as stating that the presence of dust in the flow results in effective reduction of Karman constant κ. In this instance the velocity gradients are less abrupt in the vicinity of the surface, a circumstance which facilitates the dislodging and lifting of larger quantities of dust.

For small particles of a diameter of a few microns $\omega \ll 1$. There is a limiting stationary distribution of dust in altitude, which is given by the formula

$$n(z) = n(z_1)\left(\frac{z}{z_1}\right)^{-1},$$

in which n is the dust concentration and the subscript 1 refers to a certain height near the bottom. In the case of an unlimited supply of dust on the underlying surface the flow tends toward this maximum saturation, which is the greater the smaller is ω. If the supply of dust is limited the dust concentration value at the bottom is smaller, but it tends asymptotically toward the maximum with increase in altitude z. Similar relationships are also to be expected for temperature stratified flow, although there is as yet no /92
corresponding theory[14].

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[14]Such a theory has recently been elaborated by G. I. Barenblatt and the author in a paper forwarded in the middle of 1973 for publication in the "J. Atm. Sci." One of its conclusions is that in the case of stable stratification dust concentration n(z) decreases exponentially for altitudes greater than the Monin-Obukhov scale (17.1), while in the case of free convection the value of n(z) tends toward a certain constant value with increase in z. This makes it possible to understand why the dust storms always begin near the period of maximum insolation of the surface of Mars, when the probability of occurrence of strong convection is greatest.

This property of turbulent flows of collecting and carrying within themselves a large quantity of dust represents one of the mechanisms favoring the spontaneous development of a dust storm. The propagation of dust already lifted to high altitudes, such as the altitude of the homogeneous atmosphere H, is accomplished by turbulent mixing. Let us estimate the value of the turbulent mixing coefficient required for lifting of dust to altitude H in time t. Using the formula $K \sim H^2/t$, with H = 10 km and $t = 10^5$ sec (1 day), we obtained $K \sim 10^7$ $cm^2/sec = 10^3$ m^2/sec. This value is entirely reasonable for daytime conditions on the equator, but is somewhat lower than the estimates of the turbulent mixing coefficient given by Gierasch and Goody (1968) and by the author (see Golitsyn, 1969).

Horizontal mixing is characterized by much larger coefficients. $K \sim 3 \cdot 10^{10}$ $cm^2/sec = 3 \cdot 10^6$ m^2/sec is required for the propagation of dust over 1000 km in 3 days; this value is entirely in agreement with the previous estimates of Section 18 (also see Golitsyn, 1968, 1970b). Such a value of K is an order of magnitude larger than that used in the numerical experiments by Leovy and Mintz (1969). They characterized this value as small and one can agree with this statement. However, this question also requires further study.

But let us assume that dust has been lifted above a rather extensive area. Measurements under global storm conditions (Moroz, Ksanfomality, 1972; Hanel et al., 1972; Chase et al., 1972; Kliore et al., 1972) show that during the day the surface of the planet is on the average 20-30 K cooler than in the absence of a storm, and that the vertical temperature profiles are near isothermy, owing to which the atmosphere is on the whole warmer. This is understandable if it is assumed that a large part of the solar radiation is absorbed by the dust-filled atmosphere itself. Unfortunately, the optical properties of the Martian dust are as yet unknown and only indirect estimates may be made of the absorption value (see the work by Ginzburg, 1973). In this context it is useful to cite measurements of solar radiation absorption under the conditions of the extremely dusty atmosphere in the Karakum Desert (Kondrat'yev, Vasil'yev, Grishechkin, 1971). The conditions here are characterized by the authors as those of an intense dust haze. The atmospheric absorption increases appreciably in this case, reaching approximately 20-25% of the total value of direct solar radiation.

Let us assume that for a dust cloud not as yet of very large dimensions /93
lowering of the temperature also takes place in the lower part of the cloud. Simple estimates of the amplitudes of the wind arising in this instance may be made on the basis of the data of Gierasch and Sagan (1971).

For small scales L the Rossby number $Ro = U/lL$, in which $l = 2\omega\sin\vartheta$, the Coriolis parameter, is greater than unity (the Rossby number is the measure of the ratio of the nonlinear terms in the equations of motion to the Coriolis force). The Coriolis acceleration then plays a subordinate role and may be disregarded in very rough estimates. The equation of motion with pressure as an independent variable may be written as

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial h}+\frac{\partial \varphi}{\partial x}=0, \tag{19.3}$$

$$\frac{\partial \varphi}{\partial h}=R'T, \tag{19.4}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial h}-w=0, \tag{19.5}$$

in which the role of vertical velocity is played by quantity $w = dh/dt$, $h = ln(p/p_s)$, $\varphi = gz$ is the geopotential, R' is the gas constant, and T is temperature. If temperature variation ΔT at a certain distance L is assigned, then the velocity of the motions occurring in this instance may be estimated from this system. By means of (19.5) we ascertain that the first two terms in (19.3) are of the same order. Hence it follows from (19.4) and (19.4) that

$$U^2 \sim \Delta\varphi \sim R'\Delta T\Delta h. \tag{19.6}$$

It is assumed here that the vertical and the horizontal temperature variations are of one order. Allowance for the nonstationary terms and, for example, the circular symmetry of temperature distribution introduces only additional positive terms into (19.3) of the same order as U^2L^{-1}. Let $\Delta T = 10$ K and $\Delta h = 1/2$, that is, the motions take place only up to the mean level of the atmosphere, at which the temperature in the cloud is comparable to the temperature of the remaining atmosphere (it is lower below this level and higher above it). Then, in accordance with (19.6), $U \sim (1/2R'\Delta T)^{1/2} = 30$ m/sec.

Since U is known, the restrictions on $L < U/l$ may be estimated from the condition $Ro > 1$. For latitude $\vartheta = 30°$ we have $L < 400$ km. Since the atmosphere is cooler in the lower central portion of the cloud, the motions are descending ones there, but ascending ones on the edges of the clouds. The occurrence of appreciable wind speeds and the nature of circulation should in this instance contribute toward transport of dust to the periphery of the cloud and further blowing away and lifting at its edges. The opposite picture of temperature, and hence of velocity distribution is observed at night, but because of the short duration of the summer nights it may be assumed that on the average the dust cloud will increase in a day.

According to the estimates of the dust propagation time given in the fore- /94
going, owing to horizontal turbulent mixing the cloud may reach dimensions of the order of 500 km in a few days. With larger dimensions it is necessary to take the Coriolis force into account to determine the winds, that is, the motion will be determined chiefly by the balance between the pressure gradient and the Coriolis force. With local variations in the temperature field there is also variation in the pressure field, and the wind consequently undergoes variation as well. The geostrophic wind component resulting from variation in the temperature field is termed the thermal wind. The thermal wind equations are of the form of (Gierasch, Sagan, 1971):

$$\frac{\partial u}{\partial h}=-\frac{R'}{l}\frac{\partial T}{\partial y}, \tag{19.7}$$

$$\frac{\partial v}{\partial h}=\frac{R'}{l}\frac{\partial T}{\partial x} \tag{19.8}$$

or in vectorial notation

$$\frac{\partial \mathbf{u}}{\partial h} = -\frac{R'}{l}\nabla_H T\cdot\mathbf{k}, \qquad (19.9)$$

in which k is the unit vector, which is directed upward.

It follows from these equations that the thermal wind varies with altitude and blows along the isotherms, in such a way that the cooler air remains on the right in the southern hemisphere ($l < 0$). A cyclonic vortex occurs as a result at the surface. The variation in velocity — the thermal wind — may be estimated in accordance with (19.9) a

$$\Delta U \sim \frac{R'}{l}\frac{\Delta h\Delta T}{L}. \qquad (19.10)$$

Inserting in this equation the values $\Delta h = 1/2$, $\Delta T = 30$ K, $L = 10^3$ km, and $l = 7\cdot10^{-5}$ sec^{-1}, we obtain $\Delta U \sim 40$ m/sec. An anticyclonic vortex should occur in the upper part of the troposphere, where the temperature gradient is in the opposite direction.

According to the calculations of Leovy and Mintz (1969), moderate south-easterly winds having speeds of 10-20 m/sec prevail in summer in the southern hemisphere of Mars. If our estimates are correct, the motions arising on the development of a storm should substantially alter the general wind structure.

Thus a dust cloud of sufficiently large dimensions and density, owing to the absorption of direct radiation, appreciably disrupts the temperature conditions of the Martian atmosphere, this leading to the appearance of fairly strong winds which in turn should lift additional amounts of dust into the atmosphere. It seems to us that this is one of the probable large-scale feed-back mechanisms between the wind field and the amount of dust in the atmosphere contributing toward spontaneous development of a storm and its attainment of global scales[15]. Unfortunately, until the absorption properties of the dust /95 in the range of direct solar radiation are known and a quantitative theory of the lifting of dust from the surface into the atmosphere is available — these being merely a few of the most fundamental elements of the process — any quantitative description of the process of development of a storm in time and space is inconceivable. Such a description would obviously require calculations with the largest computers.

Observations show that after a storm has reached global dimensions it dies out, that is, the dust again settles on the surface, although temporary and even repeated storm efforts are possible, such as were observed by Leo in 1924 (see Kuiper, 1961). It is not difficult to understand the need for the dying out. When a dust storm reaches the global scale the temperature contrasts in the atmosphere decrease, and the wind consequently dies down. However, if the precipitation of particles above various areas of a surface is uneven, temporary

[15]A similar idea about the nonlinear interaction of dust, radiation, and atmospheric motions leading to increase in a dust storm on Mars was also advanced independently by Gierasch and Goody (1972) and by Hess (1973).

accentuation of the temperature contrasts and intensification of the winds are again possible.

A special role must be played by the polar cap in the temperature (and wind) conditions of the atmosphere of the southern hemisphere. During this season the cap melts rapidly, in which process a large amount of heat from the atmosphere is consumed, that is, near the surface of the cap the temperature of the atmosphere must be around 150 K, the melting point of solid carbon dioxide. The great temperature difference between the atmosphere above the cap and above the regions where the ice has already melted should cause strong winds. Higher above the cap the temperature of the atmosphere may be substantially higher owing to the advection of heat and the direct heating of the atmosphere by the Sun. Observations by Hanel et al. (1972), have revealed the presence of strong temperature inversion above the southern polar region. The flow of a mass of carbon dioxide into the atmosphere on evaporation of the cap should also play a role in the process. We confine ourselves here simply to indicating the importance and complexity of these processes.

In conclusion all that remains is for the author to repeat that the establishment of any quantitative theory of the process of development and dying out of a dust storm on Mars is one of the most difficult tasks with which physics of the atmosphere has ever been faced, since allowance for too many interacting factors is required.

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